Math 520

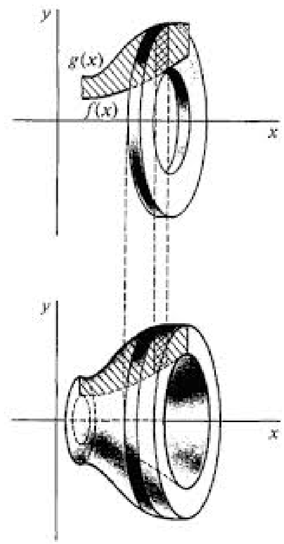
Volume: Washer Method

§6.2

A solid of revolution is an object obtained by rotating planar region about a line. Usually this line is the $x-$axis, $y-$axis, or some other line. Sometimes the resulting figure will have a cross section in the shape of a disk and other times the cross section will be washer shaped. The volume will be approximated by a sum of volumes of disks or washers which suggest the definite integral.

Volume by Washer Method

If the region between $y = f(x)$ and $y = g(x)$ is rotated about the $x-$axis, the volume may be approximated by a sum of volumes of washers.



The volume of a typical washer is

$$\pi\left[(\text{outer radius})^2(\text{thickness}) - (\text{inner radius})^2(\text{thickness})\right] = \pi[(g(x_i^*))^2 - (f(x_i^*))^2]\Delta x.$$

Thus the volume is

$$V = \int_a^b \pi[(g(x))^2 - (f(x))^2]\,dx.$$

1. Let R be the region enclosed by $y = x^2$, $y = x^3$, $x = 1$, and $x = 2$. Sketch the region. Rotate R about the $x-$axis and find the resulting volume.

Solution:

Volume of solid is= $\int_1^2 \pi[(x^3)^2 - (x^2)^2]\,dx = int_1^2\pi[(x^6 - x^4)]\,dx = \frac{418}{35}\pi \approx 37.52$

2. Let R be the region enclosed by $y = x^2$ and $y = 4x - x^2$. Sketch the region. Rotate R about the $x-$axis and find the resulting volume.

Solution:

Volume of solid is= $\int_0^2 \pi((4x - x^2))^2 - (x^2)^2\,dx = \int_0^2 \pi(16x^2 - 8x^3)\,dx = \frac{32}{3}\pi \approx 33.51$

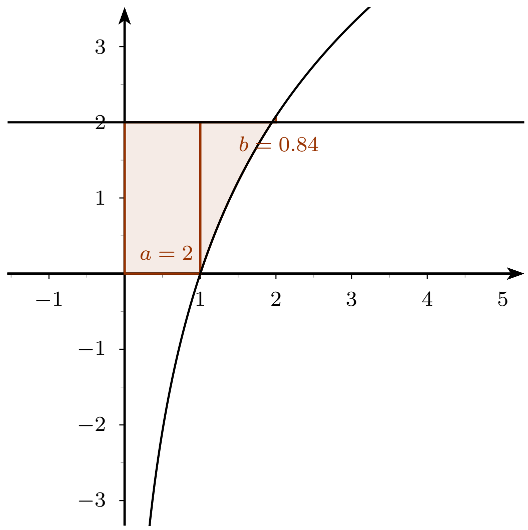
3. Let R be the region enclosed by $y = x^2$ and $y = 4x - x^2$. Sketch the region. Rotate R about the line $y = 6$ and find the resulting volume.

Solution:

Volume of solid is= $\int_0^2 \pi((6 - x^2))^2 - (6 - 4x + x^2)^2)\,dx = \int_0^2 8\pi(x^3 - 5x^2 + 6x)\,dx = \frac{64}{3}\pi \approx$ 67.02.

4. Let R be the region enclosed by $y = 3\ln x$, $y = 2$ and the $x-$ and $y-$ axes. Sketch the region. Rotate R about the $x-$axis and find the resulting volume.

Solution:



Notice we need to break up this integral because for part of the volume we can use the disc method and the other part we use the washer method.

Volume of solid is$= \displaystyle\int_0^1 \pi 2^2 \, dx + \int_1^{e^{2/3}} \pi[(2^2 - (3\ln x)^2] \, dx \approx 2.84.$

5. Let R be the region enclosed by $y = x^2 + 2$, $y = 2$, $x = 0$, and $x = 1$. Sketch the region. Rotate R about the $x-$axis and find the resulting volume.

Solution:

Volume of solid is$= \displaystyle\int_0^1 \pi[(x^2 + 1)^2 - 1)] \, dx.$