Math 520

Volume: Shell Method

§6.2

Another way to determine the definite integral for a solid of revolution is using concentric cylindrical shells. The key to this method is to visualize the solid as composed of many shells whose volumes can be determined.

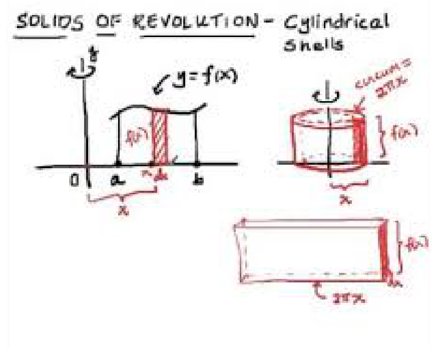
The Shell Method

The volume of the solid obtained by rotating the region bounded by $y = f(x)$, $y = 0$, $x = a$, $x = b$ about the $y-$axis may be approximated by the sum of volumes of shells. The volume of a typical shell is

$$2\pi\,(\text{radius})(\text{height})(\text{thickness}) = 2\pi x_i^* f(x_i^*)\Delta x.$$

Thus the total volume of a solid by this method of (cylindrical) shells is

$$V = \int_a^b 2\pi\, x\, f(x)\; dx.$$



1. Let R be the region bounded by $y = \sqrt{x}$, $y = 0$, $x = 1$, $x = 4$. Sketch the region. Rotate R about the $y-$axis and find the resulting volume.

Solution: Volume of the solid is $= \displaystyle\int_1^4 2\pi x^{3/2}\; dx = \frac{124\pi}{5} \approx 77.9115$

2. Let R be the region bounded by $y = x^3$, $y = 0$, $x = 0$, $x = 1$. Sketch the region. Rotate R about the $y-$axis and find the resulting volume.

Solution: Volume of the solid is $= \displaystyle\int_0^1 2\pi x^4\; dx = \frac{2\pi}{5} \approx 1.2566$

3. Let R be the region bounded by $y = e^x$, $y = 0$, $y = 3$, $x = 0$, $x = 2$. Sketch the region. Rotate R about the $x-$axis and find the resulting volume.

Solution: Using the disc method we have...
Volume of the solid is $= \displaystyle\int_0^{\ln 3} \pi(e^x)^2\; dx + \int_{\ln 3}^2 \pi 3^2\; dx \approx 38.0525$
Notice we needed to break into two integrals because the radius changed.

4. Let R be the region bounded by $y = e^x$, $y = 0$, $y = 3$, $x = 0$, $x = 2$. Sketch the region. Rotate R about the $y-$axis and find the resulting volume.

Solution: Using the shell method we have...
Volume of the solid is $= \int_0^{\ln 3} 2\pi x e^x \, dx + \int_{\ln 3}^{2} 2\pi x(3) \, dx \approx 34.4659$

5. Let R be the region bounded by $y = x^2 - 6x + 11$, $y = 0$, $x = 0$, $x = 6$. Sketch the region. Rotate R about the $y-$axis and find the resulting volume.

Solution: Using the shell method we have...
Volume of the solid is $= \int_0^6 2\pi x(x^2 - 6x + 11) \, dx = 180\pi \approx 565.487$

6. Let R be the region bounded by $y = x^2 + 1$, $y = 0$, $x = 0$, $x = 1$. Sketch the region. Rotate R about the $y-$axis and find the resulting volume.

Solution: Using the shell method we have...
Volume of the solid is $= \int_0^1 2\pi x(x^2 + 1) \, dx \approx 4.71239$

7. Let R be the region bounded by $f(x) = x^2 - 4x$, $g(x) = x$. Sketch the region. Rotate R about the $y-$axis and find the resulting volume.

Solution: Using the shell method we have...
the radius is: x
the height is: $g(x) - f(x) = x - (x^2 - 4x) = -x^2 + 5x$
Volume of the solid is $= \int_0^5 2\pi x(-x^2 + 5x) \, dx = \frac{625\pi}{6} \approx 327.249$

8. Let R be the region bounded by $y = x^3 + x + 1$, $y = 1$, $x = 1$. Sketch the region. Rotate R about $x = 2$.

Solution: Using the shell method we have...
the radius is: $2 - x$
the height is: $x^3 + x + 1 - 1 = x^3 + x$
Volume of the solid is $= \int_0^1 2\pi(2 - x)(x^3 + x + 1 - 1) \, dx \approx 6.0737$