Math 520

Volume: More Disc Method

§6.2

A solid of revolution is an object obtained by rotating planar region about a line. Usually this line is the $x-$axis, $y-$axis, or some other line. Sometimes the resulting figure will have a cross section in the shape of a disk. The volume will be approximated by a sum of volumes of disks which suggest the definite integral.

Volume by Disc Method

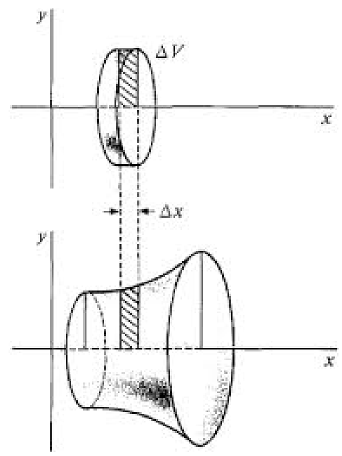
If the region under $y = f(x)$ is rotated about the $x-$axis, the volume may be approximated by a sum of volumes of disks.

The volume of a typical disk is

$$\pi(\text{radius})^2(\text{thickness}) = \pi[f(x_i^*)]^2\Delta x.$$

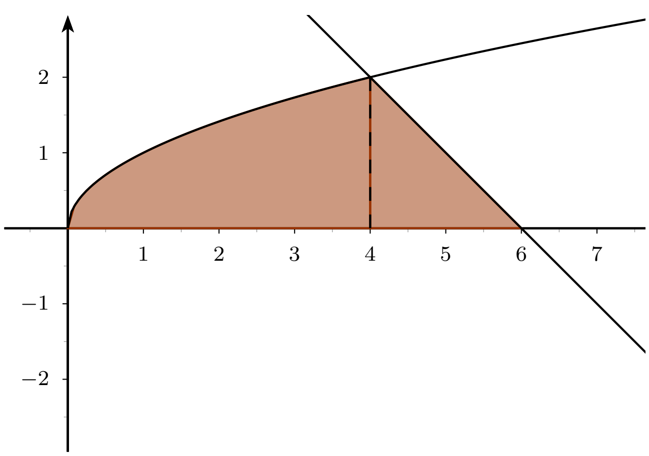
Thus the volume is

$$V = \int_a^b \pi[f(x)]^2\, dx.$$



1. Let R be the region enclosed by $y = \sqrt{x}$, $y = 6 - x$, and the $x-$axis. Sketch the region. Rotate R about the $x-$axis and find the resulting volume.

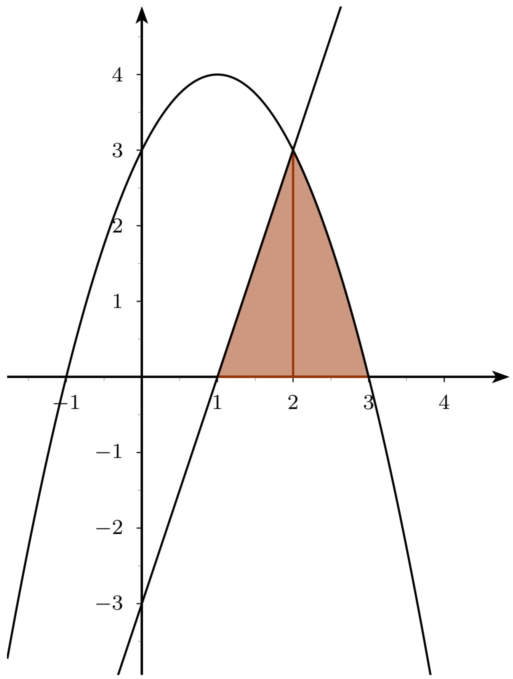
Solution: Notice this has to be separated into two integrals because the functions are different for each piece.



Volume of solid is$= \int_0^4 \pi(\sqrt{x})^2\, dx + \int_4^6 \pi(6-x)^2\, dx = \dfrac{32}{3}\pi. \approx 33.5103$

2. Let R be the region only in the first quadrant enclosed by $y = -x^2 + 2x + 3$, $y = 3x - 3$, and the $x-$axis. Sketch the region. Rotate R about the $x-$axis and find the resulting volume.

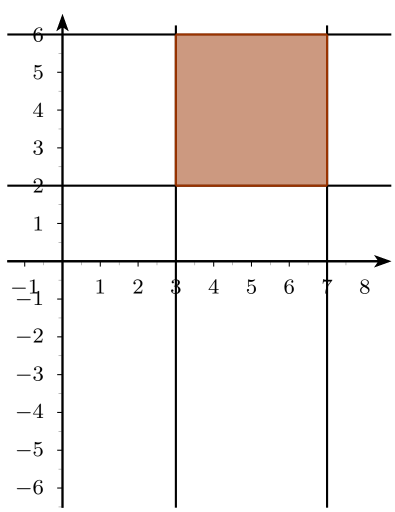
Solution: Notice this has to be separated into two integrals because the functions are different for each piece.



Volume of solid is$=\displaystyle\int_1^2 \pi(3x-3)^2\,dx+\int_2^3 \pi(-x^2+2x+3)^2\,dx=\frac{98}{15}\pi\approx 20.52507$

3. Let R be the region enclosed by $y=6$, $y=2$, $x=3$, and $x=7$. Sketch the region. Rotate R about the line $y=4$ and find the resulting volume.

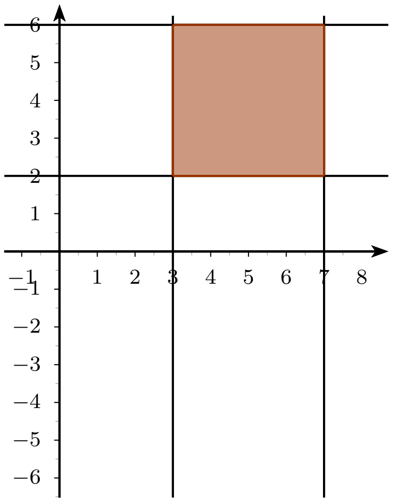
Solution: This is a cylinder



$\displaystyle\int_3^7 \pi 2^2\,dx=16\pi\approx 50.26548.$

4. Let R be the region enclosed by $y=6$, $y=2$, $x=3$, and $x=7$. Sketch the region. Rotate R about the line $y=2$ and find the resulting volume.

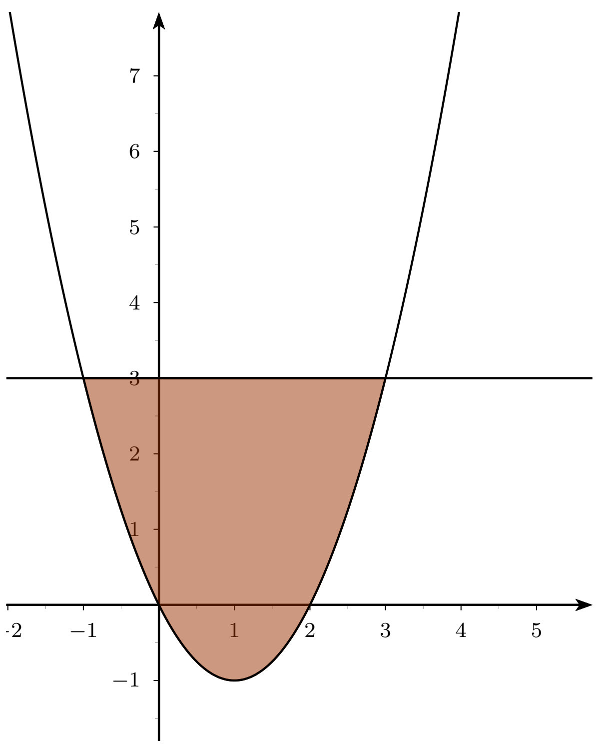
Solution: Also a cylinder



$\int_3^7 \pi 4^2 \, dx = 64\pi \approx 201.061929$

5. Let R be the region enclosed by $y = x^2 - 2x$, and $y = 3$. Sketch the region. Rotate R about the line $y = 3$ and find the resulting volume.

Solution:

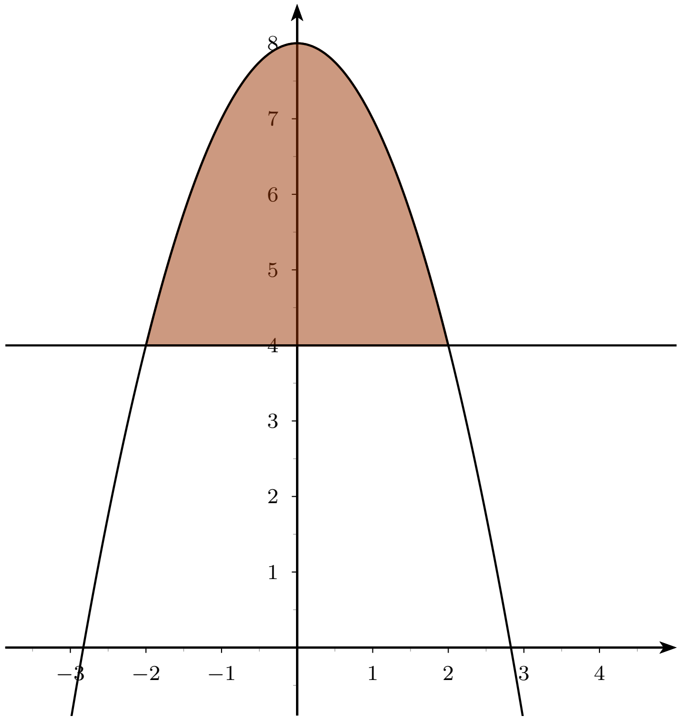


The radius of the solid is $3 - (x^2 - 2x)$. So, $A(x) = \pi r^2 = \pi(3 + 2x - x^2)^2$

Volume of the solid is $\int_{-1}^{3} \pi(3 + 2x - x^2)^2 \, dx = \frac{512}{15}\pi \approx 107.23302$

6. Let R be the region enclosed by $y = -x^2 + 8$ and $y = 4$. Sketch the region. Rotate R about the line $y = 4$ and find the resulting volume.

Solution:



The radius of the solid is $-x^2 + 8 - 4$. So,
$A(x) = \pi r^2 = \pi(-x^2 + 8 - 4)^2$.
Volume of the solid is $\int_{-2}^{2} \pi(-x^2 + 8 - 4)^2 \, dx = \frac{512}{15}\pi \approx 107.23302$