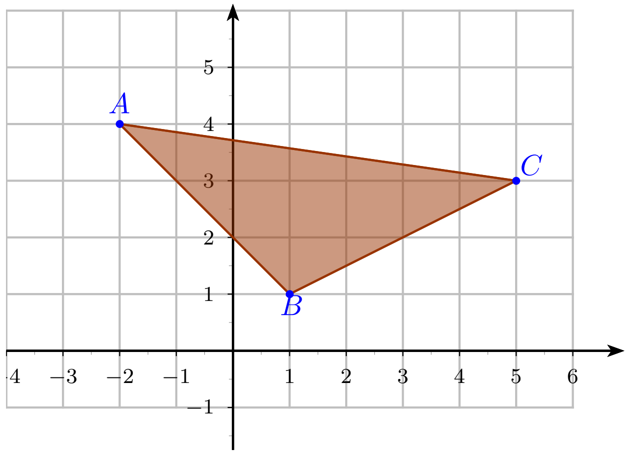
Math 520

Review

§§6.1-6.2

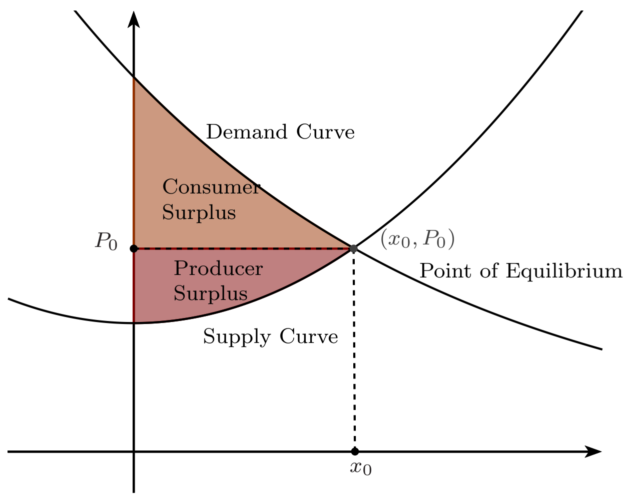
1. Find the area of the triangle using calculus.



Solution: The equations of the lines are $y = -\frac{x}{7} + \frac{26}{7}$, $y = -x + 2$, and $y = \frac{x}{2} + \frac{1}{2}$. So

$$\text{Area}= \int_{-2}^{1} [(-\frac{x}{7} + \frac{26}{7}) - (-x+2)]\ dx + \int_{1}^{5} [(-\frac{x}{7} + \frac{26}{7}) - (\frac{x}{2} + \frac{1}{2})]\ dx = 9$$

2. The consumer surplus and producer surplus are represented by the areas shown in the figure below.



If $y = 50 - .5x$ is the demand function and $y = .125x$ is the supply function, find the consumer surplus and producer surplus for the given supply and demand curves.

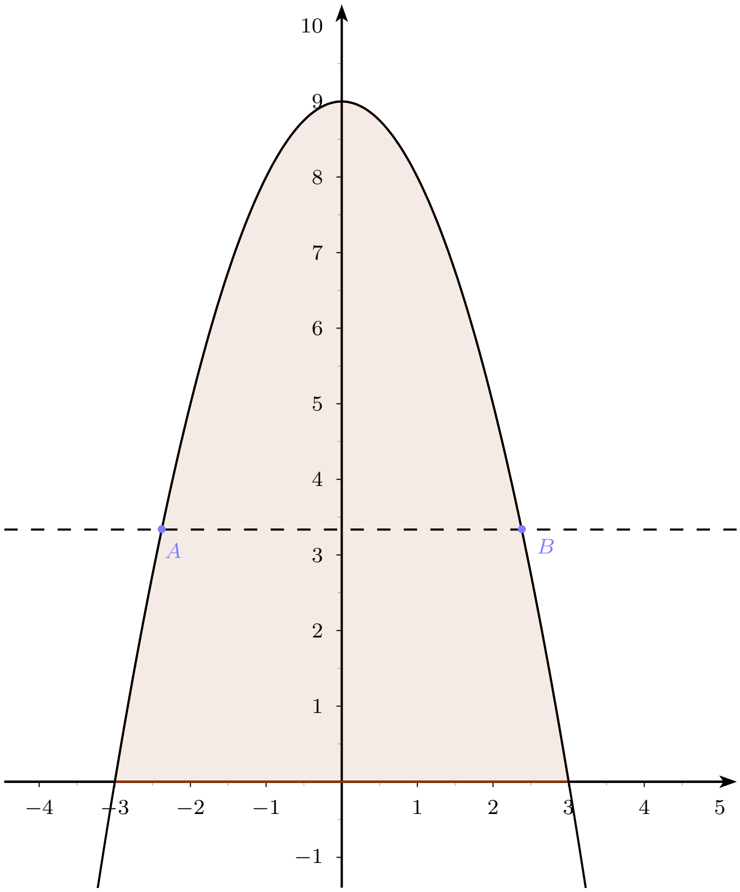
Solution: The point of equilibrium is $(80, 10)$ so

Consumer Surplus $= \int_0^{80} ((50 - .5x) - 10)\ dx = 1600.$

Producer Surplus $= \int_0^{80} (10 - .125x)\ dx = 400.$

3. Don't worry about this problem. It won't be on the test. Find b so that the line $y = b$ divides the region bounded by $y = 9 - x^2$ and $y = 0$ into two regions of equal area.

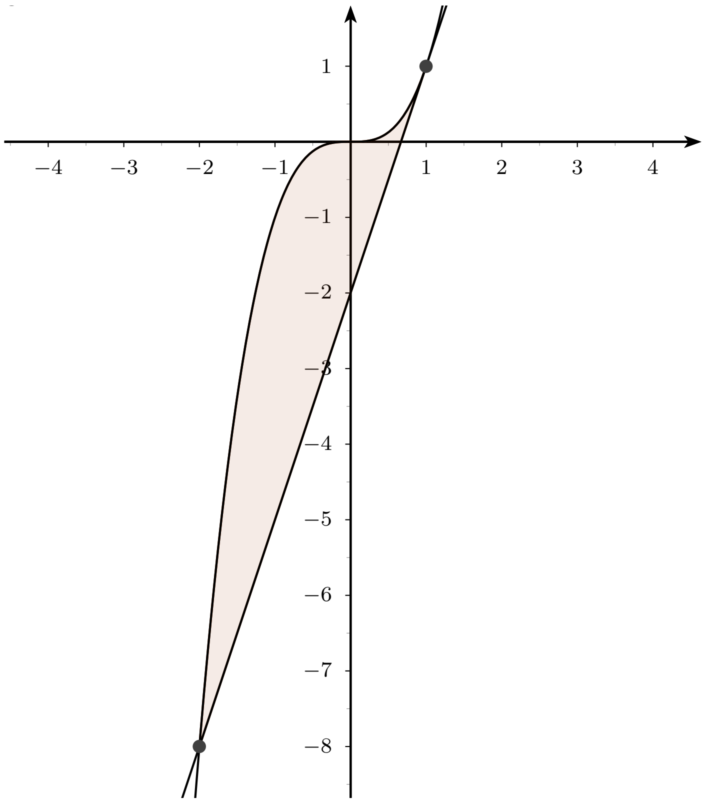
Solution:



$y = 9(1 - \frac{1}{\sqrt[3]{4}}) \approx 3.330$

4. Find the area of the region bounded by the graph $f(x) = x^3$ and the tangent line to the graph at the point $(1, 1)$.

Solution: Since $f(x) = x^3$, $f'(x) = 3x^2$, and $f'(1) = 3$. The equation of the tangent line is $y = 3x - 2$. The $x-$coordinates of the points of intersection of the tangent line and the function are $x = -2$ and 1.

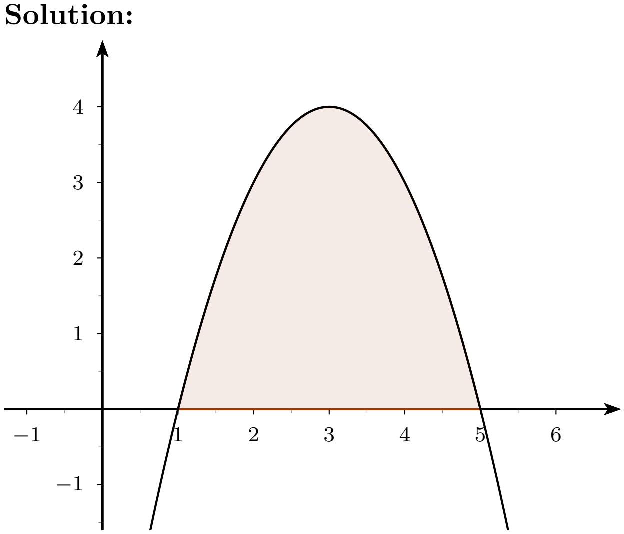


Therefore, the area is

$$\int_{-2}^{1} (x^3 - (3x - 2))]\, dx = \frac{27}{4}$$

5. Find the volume of the solid generated by revolving the region bounded by the graphs of $y = -x^2 + 6x - 5$ and $y = 0$ about the $x-$axis.

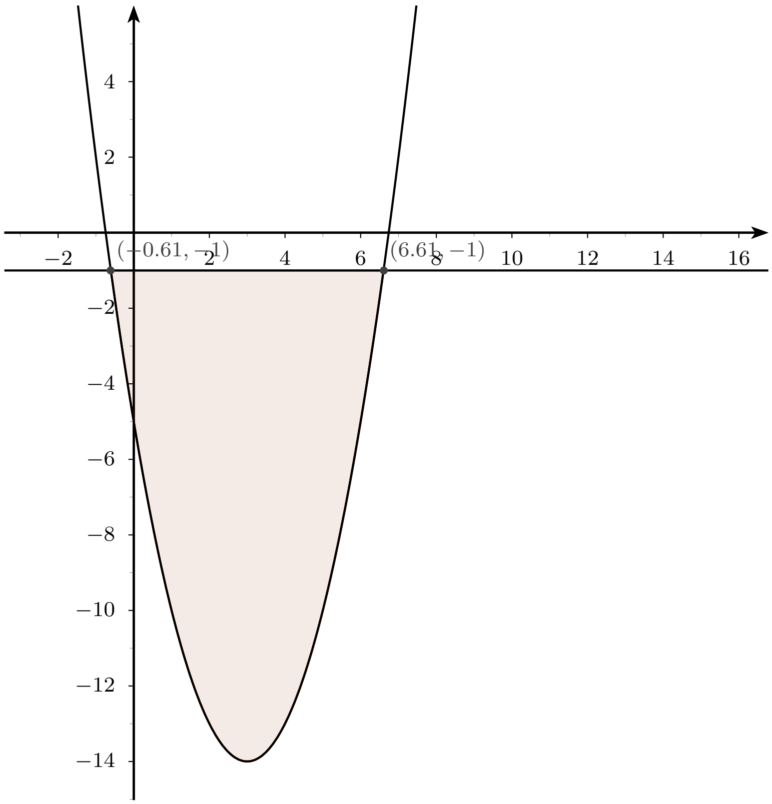
Solution:



$$V = \int_{1}^{5} \pi(-x^2 + 6x - 5)^2\, dx = \frac{512}{15}\pi$$

6. Find the volume of the solid generated by revolving the region bounded by the graphs of $y = x^2 - 6x - 5$ and $y = -1$ about the line $y = -1$.

Solution:



$$V = \int_{-.61}^{6.61} \pi(x^2 - 6x - 5 - (-1))^2\, dx = 2041.911791$$