Math 520

Surface Area

§6.3+

The same technique we used integrals to calculate the length of a curve (arc length). As with other integral applications, arc length will be the limit of the sums of approximations of the lengths of small pieces of the curve.

> **Surface Area**
>
> If $y = f(x)$ has a continuous derivative on $[a, b]$, then the surface area, S, of the solid formed by rotating the graph of f about a horizontal or vertical axis is
>
> $$S.A. = \int_a^b 2\pi\, r(x)\, \sqrt{1 + (f'(x))^2}\, dx$$
>
> where $r(x)$ is the distance between the graph of f and the axis of rotation.

1. For each of the following, find the arc length of the graph of the given function over the interval $\left[\frac{\pi}{4}, \frac{\pi}{3}\right]$.

   (a) $y = \sin x$

   Solution: $\int_{\pi/4}^{\pi/3} \sqrt{1 + \cos^2(x)}\, dx \approx .307$

   (b) $y = \cos x$

   Solution: $\int_{\pi/4}^{\pi/3} \sqrt{1 + \sin^2(x)}\, dx \approx .334$

   (c) $y = \tan x$

   Solution: $\int_{\pi/4}^{\pi/3} \sqrt{1 + \sec^4(x)}\, dx \approx .779$

   (d) $y = \sec x$

   Solution: $\int_{\pi/4}^{\pi/3} \sqrt{1 + \sec^2(x)\tan^2(x)}\, dx \approx .645$

   (e) $y = \csc x$

   Solution: $\int_{\pi/4}^{\pi/3} \sqrt{1 + \csc^2(x)\cot^2(x)}\, dx \approx .371$

   (f) $y = \cot x$

   Solution: $\int_{\pi/4}^{\pi/3} \sqrt{1 + \csc^4(x)}\, dx \approx .498$

2. For the functions and intervals in #1, compute the surface area of the solid generated by rotating the curve about the $x-$axis.

Solution:

(a) $\displaystyle\int_{\pi/4}^{\pi/3} 2\pi \sin(x)\sqrt{1+\cos^2(x)}\, dx \approx 1.521$

(b) $\displaystyle\int_{\pi/4}^{\pi/3} 2\pi \cos(x)\sqrt{1+\sin^2(x)}\, dx \approx 1.271$

(c) $\displaystyle\int_{\pi/4}^{\pi/3} 2\pi \tan(x)\sqrt{1+\sec^4(x)}\, dx \approx 6.663$

(d) $\displaystyle\int_{\pi/4}^{\pi/3} 2\pi \sec(x)\sqrt{1+\sec^2(x)\tan^2(x)}\, dx \approx 6.884$

(e) $\displaystyle\int_{\pi/4}^{\pi/3} 2\pi \csc(x)\sqrt{1+\csc^2(x)\cot^2(x)}\, dx \approx 2.973$

(f) $\displaystyle\int_{\pi/4}^{\pi/3} 2\pi \cot(x)\sqrt{1+\csc^4(x)}\, dx \approx 2.455$