Math 520

Slope Fields

§7.2

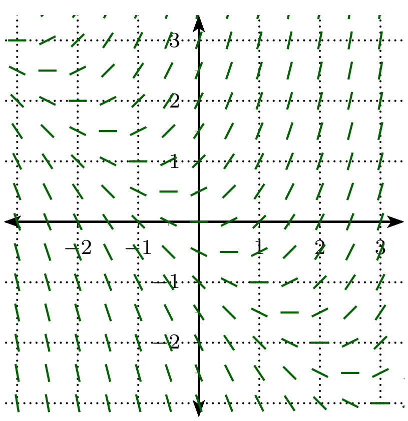
Slope, or direction fields are the graphical representation of a differential equation. They can give a picture of a differential equation. This is often useful when the solution algebraically is unobtainable.

Example: Sketch the graph of the solution to $y' = x + y$ that satisfies the initial condition $y(0) = 1$.

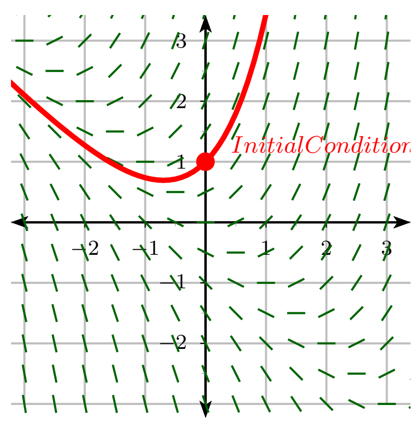
Solution: First we will plot the slope field. To do this we need to find the slope of the solution curve at every point on a grid. To organize the data, complete a table like the one below.

x	-3	-2	-1	0	1	2	3	-3	-2	-1	0	1	2	3	-3	$\dots$
y	-3	-3	-3	-3	-3	-3	-3	-2	-2	-2	-2	-2	-2	-2	-1	$\dots$
$y' = x + y$																

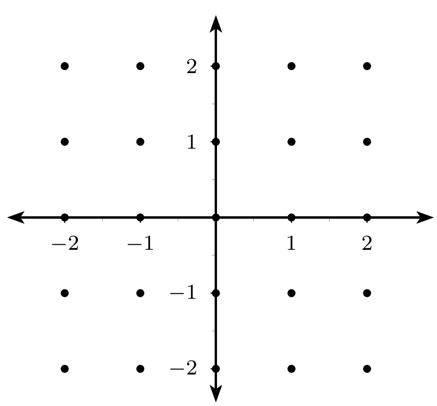
Using the table as our guide, we draw short line segments at a number of points on our grid, (x, y) with slope $x + y$. For instance the line segment at the point $(-3, -3)$ is -6. To get more detail, estimate the slope of the short line segments at points between grid dots.



Finally, we can now sketch the solution curve through the point $(0, 1)$, our initial condition, by following the direction field. One hint is to draw the curve so that it is parallel to nearby line segments.

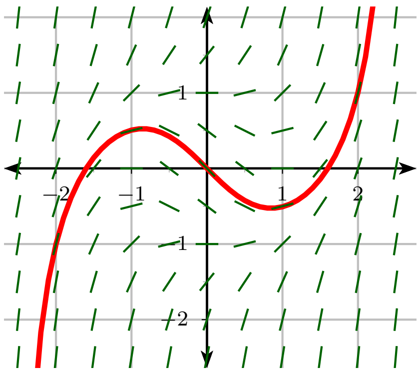


1. (a) Sketch the direction field for the differential equation $y' = x^2 + y^2 - 1$.

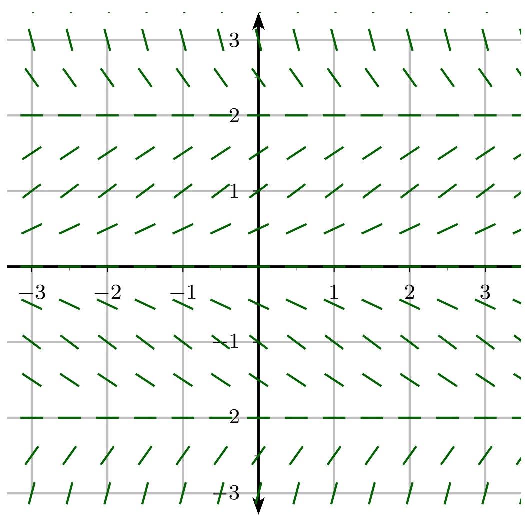


(b) Use part (a) to sketch the solution curve whose initial condition is $y(0) = 0$.

Solution:



2. A direction field for the differential equation $y' = y(1 - \frac{1}{4}y^2)$ is shown below.



(a) Sketch the graphs of the solutions that satisfy the given initial conditions.

i. $y(0) = 1$

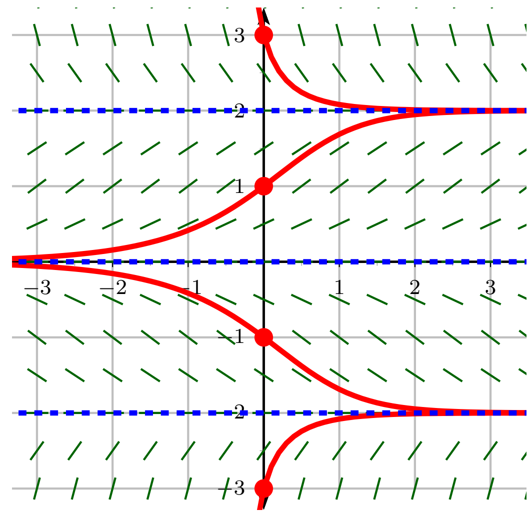
ii. $y(0) = -1$

iii. $y(0) = -3$

iv. $y(0) = 3$

(b) Draw lines representing all the equilibrium solutions.

Solution:



3. Match the differential equation with its direction field.

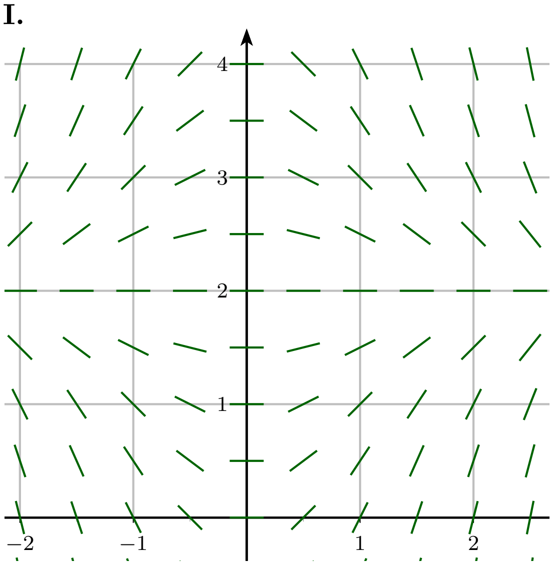
   (a) $y' = 2 - y$

   (b) $y' = x(2 - y)$

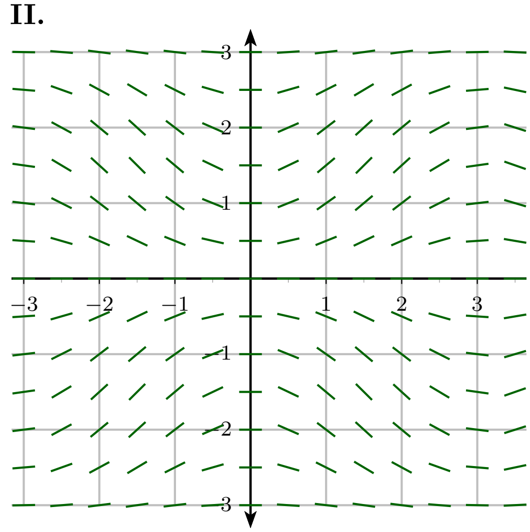
   (c) $y' = x + y - 1$

   (d) $y' = \sin x \sin y$

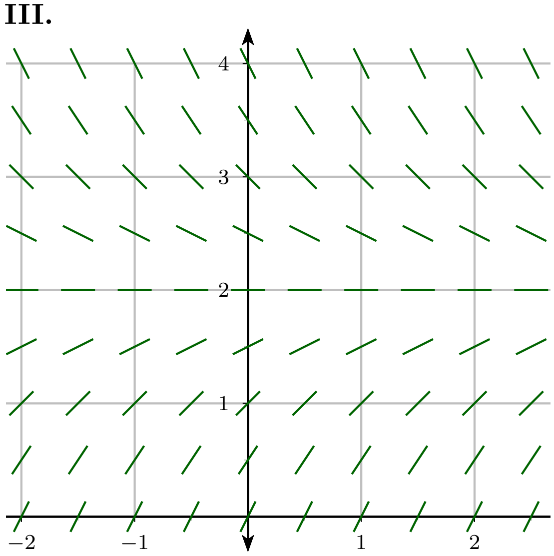
I.



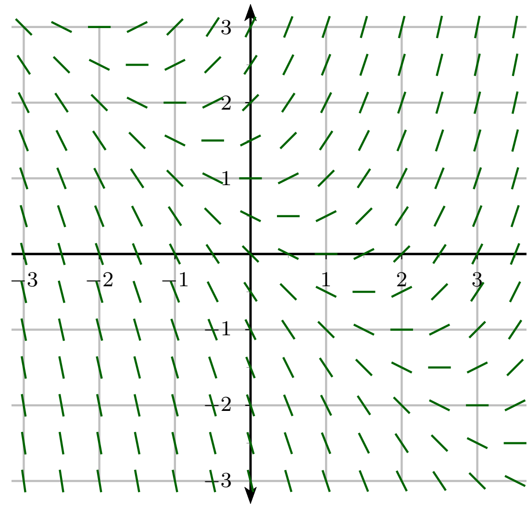
II.



III.



IV.



4. Use the direction field labeled **II.** (above) to sketch the graphs of the solutions that satisfy the initial conditions.

   (a) $y(0) = 1$

   (b) $y(0) = 2$

   (c) $y(0) = -1$

Solution: