Math 520

Shapes of Curves

§4.3

This section ties in three different tests we can apply to functions to determine their shapes. The tests are 1) The Closed Interval Test, 2) First Derivative Test, and 3) Second Derivative Test. Note that the second derivative test here is different than the one in your book.

How to Find Absolute Extrema: The Closed Interval Method

The absolute extreme values of a continuous function f on a closed interval $[a, b]$ always exist.
The occur either at a, at b, or at a critical number of f in (a, b).

To find absolute maximum and minimum values of f on a closed interval $[a, b]$, use the **Closed Interval Method**.

1. Find $f(x)$ at the critical numbers x in (a, b).
2. Find $f(a)$ and $f(b)$.
3. Choose the largest and smallest values from the results of steps 1 and 2.

How to find Local Extrema: The First Derivative Test

This test is based on the **Increasing/Decreasing Test** which states that

If $f'(x) < 0$, then f is decreasing
If $0 < f'(x)$, then f is increasing

To find local maximum and minimum values of f on a closed interval $[a, b]$, use the **First Derivative Test**.
Suppose that c is a critical number of a continuous function f.

- If the sign of f' changes from $+$ to $-$ at c, then f has a local maximum at c.
- If the sign of f' changes from $-$ to $+$ at c, then f has a local minimum at c.
- If the sign of f' does not change at c, then f has no local maximum or minimum at c.

How to find Inflection Points: The Second Derivative Test

This test is based on the **Concavitity Test** which states that

If $f''(x) < 0$, then f is concave down
If $0 < f''(x)$, then f is concave up

To find the points of inflection of f on a closed interval (a, b), use the **Second Derivative Test**.
Suppose that k is a kritical number of a continuous function f, this means

$$f''(k) = 0 \quad \text{or} \quad f''(k) \text{ is undefined}$$

- If the sign of f'' changes from $+$ to $-$ at c, then f has a point of inflection at c.
- If the sign of f'' changes from $-$ to $+$ at c, then f has a point of inflection at c.
- If the sign of f'' does not change at c, then f has no point of inflection at c.

1.