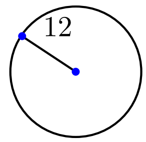
Math 520

More Related Rates

§4.1

1. A rock is dropped into a pond causing a series of circular wavefront of ripples whose radius increases at 3 inches per second. How fast is the area of the circle of ripples expanding at the instant that the circle has radius of 12 inches.

Solution:



The equation of the area of a circle is: $A = \pi r^2$

Because the ripples are expanding, **both the radius and area are really functions of time**. Differentiate the area formula with respect to time, t.

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

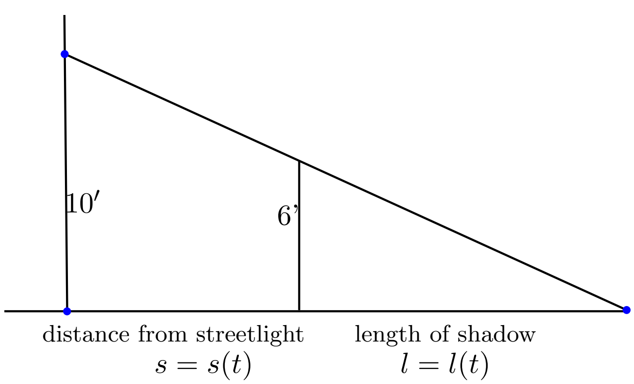
Substituting all known quantities into this equation and solve for the desired rate.

We are given the when $r = 12$, $\frac{dr}{dt} = 3$, thus,

$$\frac{dA}{dt} = 2\pi(12)(3)$$
$$\frac{dA}{dt} = 72\pi$$

2. Jo is 6 feet tall and walking away from a 10-foot streetlight, at the rate of 3 feet per second. As he walks away from the streetlight, his shadow gets longer. How fast is the length of Matt's shadow increasing when he is 8 feet from the streetlight?

Solution: First we'll draw a picture and introduce notation.



To find a relationship between these two rates, we need to find a relationship between s and l. Using similar triangles we have

$$\frac{10}{s+l} = \frac{6}{l}.$$

Simplifying this equation before differentiating makes this problem more simple. So,

$$\begin{aligned}\frac{10}{s+l} &= \frac{6}{l}\\ 10l &= 6(s+l)\\ 4l &= 6s\end{aligned}$$

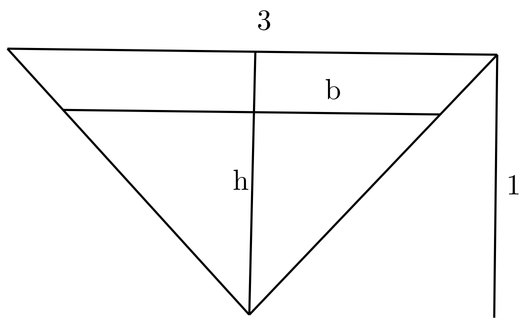
Differentiating both sides yields,

$$4\frac{dl}{dt} = 6\frac{ds}{dt}$$

Substituting the given information into the resulting equation gives us $4\frac{dl}{dt} = 6(3)$. So, $\frac{dl}{dt} = 4.5$. Interestingly, we didn't use the fact that Jo is 8 feet from the streetlight. This means that the rate of change of the length of the shadow is the same regardless of the distance Jo is from the streetlight.

3. A trough in the form of a right isosceles prism is 10 feet long. The ends have the shape of isosceles triangles that are 3 feet across at the top and have a height of 1 foot. If the trough is being filled with water at a rate of 12 cubic feet per minute, how fast is the water level rising when the water is .5 feet deep?

Solution:



The trough if in the shape of a right triangular prism. The volume of any right prism is $(areabase) \cdot (height)$. Since we know $l = 10$,

$$V = \frac{1}{2}b(h)(10)$$

We need to find a relationship between b and h. Using similar triangles we have

$$\frac{3}{1} = \frac{b}{h}.$$

Simplifying this equation before differentiating makes this problem more simple. So, $b = 3h$ and

$$V = \frac{1}{2}3h(h)(10) = 15h^2$$

Differentiating both sides yields,

$$\frac{dV}{dt} = 30h\frac{dh}{dt}$$

Substituting the given information into the resulting equation gives us $12 = 30\left(\frac{1}{2}\right)\frac{dh}{dt}$. So, $\frac{dh}{dt} = \frac{4}{5}$.