Math 520

Newton's Slide

§4.8

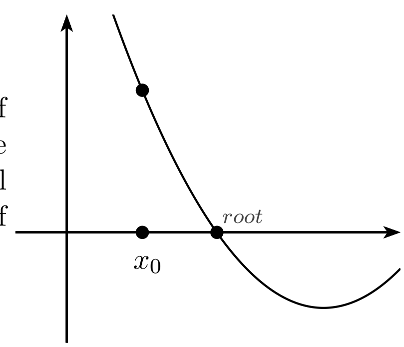
Suppose you make a guess at the solution to $f(x) = 0$. Newton's method can be used to obtain a more accurate guess from your initial guess.

Newton's Method

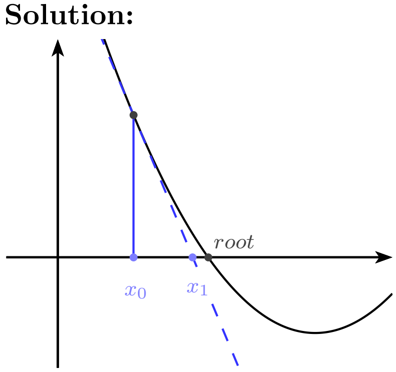
Let f be a continously differentiable function on an open interval with a real root. If x_0 is an estimate of the root, then to find another (and hopefully better) approximation of the root calculate...

$$\begin{aligned} x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\ x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} \\ x_3 &= x_2 - \frac{f(x_2)}{f'(x_2)} \\ &\vdots \\ x_{n+1} &= x_n - \frac{f(x_n)}{f'(x_n)} \end{aligned}$$

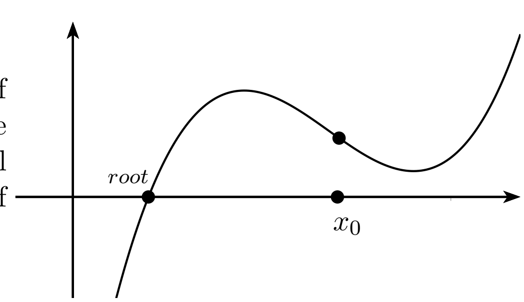
1. Using the graph at the right, if the initial guess of the root of f is given as x_0, draw the tangent line used to find a better guess, x_1, of the root. Label the point x_1. Is this a better approximation of the root or not?



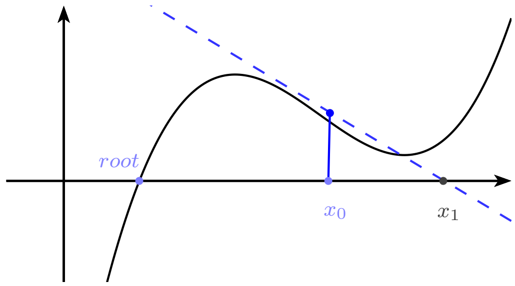
Solution:



2. Using the graph at the right, if the initial guess of the root of f is given as x_0, draw the tangent line used to find a better guess, x_1, of the root. Label the point x_1. Is this a better approximation of the root or not?



Solution:



3. Use Newton's method with $x_0 = 1$ to estimate a root of $f(x) = 2x^3 - x - 2$ accurate to 5 digits.

Solution:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{2x_n^3 - x_n - 2}{6x_n^2 - 1}$$

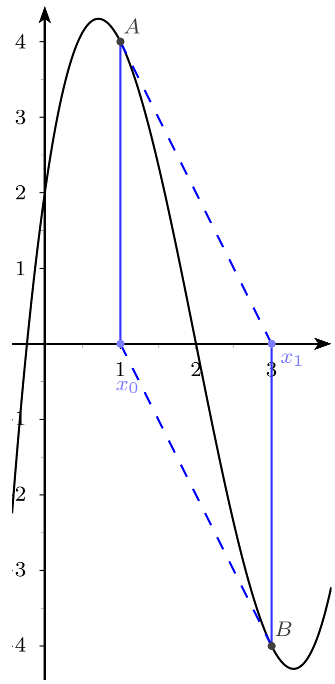
With $x_0 = 1$ using your calculator we have
$x_1 = 1.12$
$x_2 \approx 1.1664921$
$x_3 \approx 1.1653743$
$x_4 \approx 1.1653730$
Because the first 5 digits after the decimal point appear stable, $r \approx 1.16537$ is a reasonable estimate to a root, accurate to 5 digits.

4. Use Newton's method with $x_0 = 1$ to estimate a root of $f(x) = x^3 - 6x^2 + 7x + 2$ accurate to 5 digits.

Solution:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^3 - 6x_n^2 + 7x_n + 2}{3x_n^2 - 12x_n + 7}$$

With $x_0 = 1$ using your calculator we have
$x_1 = 3$
$x_2 = 1$
$x_3 = 3$
$x_4 = 1$
Looks like we are stuck in an endless cycle. Why? Can you tell using the figure below?



A
B
4
3
2
1
-1
-2
-3
-4
1
2
3
x_0
x_1