Math 520

Intro to Differential Equations

§7.1

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1. Show that $y = 2x^2$ is a solution of the differential equation $y' - 4x = 0$.

Solution: $y' = 4x$. So, $y' - 4x = 4x - 4x = 0$

2. Show that $y = x - x^{-1}$ is a solution of the differential equation $xy' + y = 2x$.

Solution: $y' = 1 + \frac{1}{x^2}$, So,

$$\begin{aligned} xy' + y &= x(1 + \frac{1}{x^2}) + x - \frac{1}{x} \\ &= x + \frac{1}{x} + x - \frac{1}{x} \\ &= 2x \end{aligned}$$

3. Show that $y = c_2 x + c_1 - \dfrac{2x^3}{3}$, where c_1 and c_2 are constants, is a solution of the second order differential equation $y'' + 4x = 0$.

Solution: $y' = c_2 - 2x^2$ and $y'' = -4x$. So,

$$\begin{aligned} y'' + 4x &= -4x + 4x \\ &= 0 \end{aligned}$$

4. For what values of r does the function $y = e^{rx}$ satisfy the differential equation $2y'' + y' - y = 0$?

Solution: $y' = re^{rx}$ and $y'' = r^2 e^{rx}$. So,

$$\begin{aligned} 2y'' + y' - y &= 2r^2 e^{rx} + r^{rx} - e^{rx} \\ &= e^{rx}(2r^2 + r - 1) \end{aligned}$$

So, $e^{rx} = 0$ or $2r^2 + r - 1 = 0$. Since e^{rx} will never equal 0 we need to solve

$$\begin{aligned} 2r^2 + r - 1 &= 0 \\ (2r - 1)(r + 1) &= 0 \end{aligned}$$

So $r = \frac{1}{2}$ or $r = -1$.

5. The general solution to the differential equation $y' = 2xy$ is $y = Ce^{x^2}$ where $C > 0$. Find a solution that satisfies the initial condition $y(1) = 3$.

Solution: Since $y(1) = 3$, we need $3 = Ce^{1^2}$. Thus, $C = \frac{3}{e}$.
So the solution is $y = \dfrac{3}{e}e^{x^2} = 3e^{x^2-1}$

6. Consider the differential equation $y' = x^2y$.
   (a) Show that every member of the family of functions $y = Ce^{x^3/3}$ is a solution.

   Solution: $y' = x^2(Ce^{x^3/3}) = x^2 = 2y$

   (b) Find a solution to the above differential equation that satisfies the initial condition $y(0) = 8$.

   Solution: Since, $y(0) = 8$ we have

$$8 = Ce^{0^3/3}$$
$$8 = Ce^0$$
$$8 = C$$

   So the soution is $y = 8e^{x^3/3}$

   (c) Find a solution to the above differential equation that satisfies the initial condition $y(8) = 0$.

   Solution: Since, $y(8) = 0$, we have $0 = Ce^{8^3/3}$. So, $C = 0$. The solution is $y = 0$.