Math 520

Fundamental Theorem of Calculus: Part 1

§5.3

The Fundamental Theorem of Calculus relates derivatives and integrals.

The Fundamental Theorem of Calculus: Part 1

If f is a continuous function on $[a,b]$ and F is any antiderivative of f, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Table of Indefinite Ingegrals	
$\int cf(x) = c\int f(x) + C$	$\int \sin x\,dx = -\cos x + C$
$\int (f(x)+g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$	$\int \sec^2 x = \tan x + C$
$\int x^n\,dx = \int \frac{x^{n+1}}{n+1} + C\ (n \neq -1)$	$\int \sec x \tan x = \sec x + C$
$\int \frac{1}{x} = \ln\lvert x\rvert + C$	$\int \frac{1}{\sqrt{1-x^2}} = \sin^{-1} x + C$
$\int \cos x\,dx = \sin x + C$	$\int \frac{1}{1+x^2} = \tan^{-1} x + C$

1. Evaluate the following definate integrals.

   (a) $\int_1^3 (6x^2 + 2x)\,dx$

   Solution: $\int_1^3 (6x^2+2x)\,dx = 2x^3 + x^2\Big|_1^3 = [2(3)^3 + 3^2] - [2(1)^2 + 1^2] = 60$

   (b) $\int_0^1 \sqrt{x}\,dx$

   Solution: $\int_0^1 \sqrt{x}\,dx = \frac{x^{3/2}}{3/2} = \frac{2}{3}x^{3/2}\Big|_0^1 = \frac{2}{3}\left(1^{3/2} - 0^{3/2}\right) = \frac{2}{3}$

   (c) $\int_{-\pi}^{\pi} \sin x\,dx$

   Solution: $\int_{-\pi}^{\pi} \sin x\,dx = -\cos x\Big|_{-\pi}^{\pi} = -\cos\pi + \cos(-\pi) = 0$

   (d) $\int_1^3 \frac{1}{x}\,dx$

Solution: $\ln 3$

(e) $\displaystyle\int_{-1}^{1} \frac{1}{x^2}\,dx$

Solution: The Fundamental Theorem doesn't apply to this because the function is not continuous on $[-1, 1]$. We will learn how to do this later.

(f) $\displaystyle\int_{0}^{1} \frac{1}{1+x^2}\,dx$

Solution: $\displaystyle\int_{0}^{1} \frac{1}{1+x^2}\,dx = \tan^{-1} 1 - \tan^{-1} 0 = \frac{\pi}{4}$

Displacement and Distance

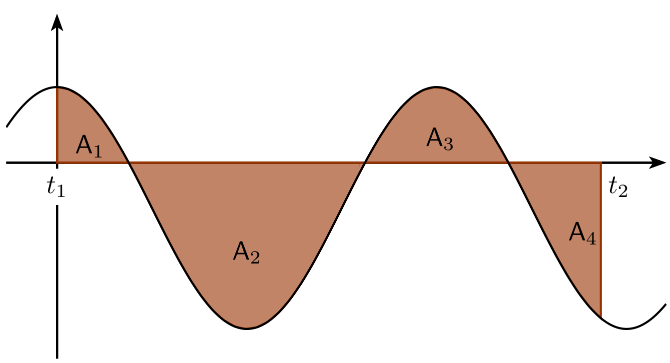
If an object moves along a straight line with position function $s(t)$, then its velocity is $v(t) = s'(t)$ so

$$\int_{t_1}^{t_2} v(t)\;dt = s(t_2) - s(t_1)$$

is the net change of position, or **displacement**, of the particle during time period from t_1 to t_2. Also,

$$\int_{t_1}^{t_2} |v(t)|\;dt$$

is the **total distance traveled** of the particle during time period from t_1 to t_2.



displacement

$$= \int_{t_1}^{t_2} v(t)\;dt = A_1 - A_2 + A_3 - A_4$$

distance

$$= \int_{t_1}^{t_2} |v(t)|\;dt = A_1 + A_2 + A_3 + A_4$$

2. A particle moves along a line so that its velocity at time t is

$$v(t) = t^2 - t - 6$$

measure in meters per second.

(a) Find the displacement of the particle during the time period $1 \le t \le 4$.

Solution: $\displaystyle\int_{1}^{4} v(t)\,dt = \int_{1}^{4} (t^2 - t - 6)\;dt = \frac{t^3}{3} - \frac{t^2}{2} - 6t\Big|_{1}^{4} = -\frac{9}{2}$

(b) Find the distance traveled during this time period.

Solution: $v(t) \leq 0$ on $[1, 3]$ and $0 \leq v(t)$ on $[3, 4]$ So,

$$\begin{aligned}
\int_1^4 |v(t)|\ dt &= \int_1^3 (-v(t))\ dt + \int_3^4 v(t)\ dt \\
&= \int_1^3 (-t^2 + t + 6)\ dt + \int_3^4 (t^2 - t + 6)\ dt \\
&= \frac{61}{6}
\end{aligned}$$