1. (a) $\frac{d}{dx}(xy + 2x + 3x^2) = \frac{d}{dx}(4) \Rightarrow (x \cdot y' + y \cdot 1) + 2 + 6x = 0 \Rightarrow xy' = -y - 2 - 6x \Rightarrow$

$y' = \frac{-y-2-6x}{x}$ or $y' = -6 - \frac{y+2}{x}$.

(b) $xy + 2x + 3x^2 = 4 \Rightarrow xy = 4 - 2x - 3x^2 \Rightarrow y = \frac{4-2x-3x^2}{x} = \frac{4}{x} - 2 - 3x$, so $y' = -\frac{4}{x^2} - 3$.

(c) From part (a), $y' = \frac{-y-2-6x}{x} = \frac{-(4/x - 2 - 3x) - 2 - 6x}{x} = \frac{-4/x - 3x}{x} = -\frac{4}{x^2} - 3$.

2. (a) $\frac{d}{dx}(4x^2 + 9y^2) = \frac{d}{dx}(36) \Rightarrow 8x + 18y \cdot y' = 0 \Rightarrow y' = -\frac{8x}{18y} = -\frac{4x}{9y}$

(b) $4x^2 + 9y^2 = 36 \Rightarrow 9y^2 = 36 - 4x^2 \Rightarrow y^2 = \frac{4}{9}(9 - x^2) \Rightarrow y = \pm\frac{2}{3}\sqrt{9 - x^2}$, so

$y' = \pm\frac{2}{3} \cdot \frac{1}{2}(9 - x^2)^{-1/2}(-2x) = \mp\frac{2x}{3\sqrt{9 - x^2}}$

(c) From part (a), $y' = -\frac{4x}{9y} = -\frac{4x}{9\left(\pm\frac{2}{3}\sqrt{9 - x^2}\right)} = \mp\frac{2x}{3\sqrt{9 - x^2}}$.

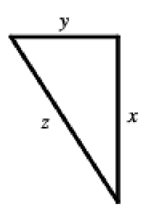
3. $\frac{d}{dx}(x^3 + x^2y + 4y^2) = \frac{d}{dx}(6) \Rightarrow 3x^2 + (x^2y' + y \cdot 2x) + 8yy' = 0 \Rightarrow x^2y' + 8yy' = -3x^2 - 2xy \Rightarrow$

$(x^2 + 8y)y' = -3x^2 - 2xy \Rightarrow y' = -\frac{3x^2 + 2xy}{x^2 + 8y} = -\frac{x(3x + 2y)}{x^2 + 8y}$

6. $x^2 + y^2 = 25 \Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow x\frac{dx}{dt} = -y\frac{dy}{dt} \Rightarrow \frac{dx}{dt} = -\frac{y}{x}\frac{dy}{dt}$.

When $y = 4$, $x^2 + 4^2 = 25 \Rightarrow x = \pm 3$. For $\frac{dy}{dt} = 6$, $\frac{dx}{dt} = -\frac{4}{\pm 3}(6) = \mp 8$.

13.



We are given that $\frac{dx}{dt} = 60$ mi/h and $\frac{dy}{dt} = 25$ mi/h. $z^2 = x^2 + y^2 \Rightarrow$

$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} \Rightarrow z\frac{dz}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt} \Rightarrow \frac{dz}{dt} = \frac{1}{z}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$.

After 2 hours, $x = 2(60) = 120$ and $y = 2(25) = 50 \Rightarrow z = \sqrt{120^2 + 50^2} = 130$,

so $\frac{dz}{dt} = \frac{1}{z}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right) = \frac{120(60) + 50(25)}{130} = 65$ mi/h.

44. $f(x) = \frac{x}{x^2 + 4}$, $[0, 3]$. $f'(x) = \frac{(x^2 + 4)1 - x(2x)}{(x^2 + 4)^2} = \frac{4 - x^2}{(x^2 + 4)^2} = 0 \Leftrightarrow x = \pm 2$, but -2 is not in the interval $[0, 3]$.

$f(0) = 0$, $f(2) = \frac{1}{4} = 0.25$, $f(3) = \frac{3}{13} \approx 0.23$. So $f(2) = \frac{1}{4}$ is the absolute maximum and $f(0) = 0$ is the absolute minimum.

3. (a) Use the Increasing/Decreasing (I/D) Test.

(b) Use the Concavity Test.

(c) At any value of x where the concavity changes, we have an inflection point at $(x, f(x))$.

6. (a) f is increasing on the intervals where $f'(x) > 0$, namely, $(2, 4)$ and $(6, 9)$.

(b) f has a local maximum where it changes from increasing to decreasing, that is, where f' changes from positive to negative (at $x = 4$). Similarly, where f' changes from negative to positive, f has a local minimum (at $x = 2$ and at $x = 6$).

(c) When f' is increasing, its derivative f'' is positive and hence, f is concave upward. This happens on $(1, 3)$, $(5, 7)$, and $(8, 9)$. Similarly, f is concave downward when f' is decreasing—that is, on $(0, 1)$, $(3, 5)$, and $(7, 8)$.

(d) f has inflection points at $x = 1, 3, 5, 7$, and 8, since the direction of concavity changes at each of these values.

23. (a) $A(x) = x\sqrt{x+3} \quad\Rightarrow\quad A'(x) = x \cdot \frac{1}{2}(x+3)^{-1/2} + \sqrt{x+3} \cdot 1 = \dfrac{x}{2\sqrt{x+3}} + \sqrt{x+3} = \dfrac{x + 2(x+3)}{2\sqrt{x+3}} = \dfrac{3x+6}{2\sqrt{x+3}}$.

The domain of A is $[-3, \infty)$. $A'(x) > 0$ for $x > -2$ and $A'(x) < 0$ for $-3 < x < -2$, so A is increasing on $(-2, \infty)$ and decreasing on $(-3, -2)$.

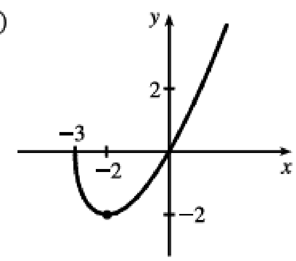
(b) $A(-2) = -2$ is a local minimum value.

(c) $$A''(x) = \frac{2\sqrt{x+3} \cdot 3 - (3x+6) \cdot \dfrac{1}{\sqrt{x+3}}}{\left(2\sqrt{x+3}\right)^2}$$

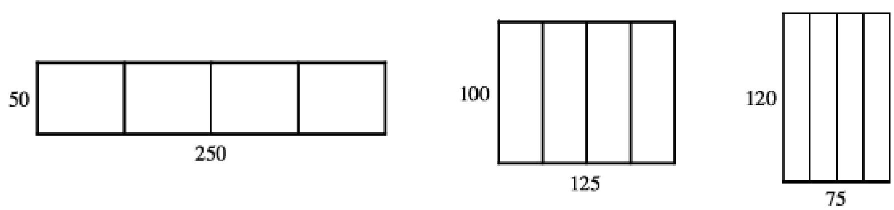
$$= \frac{6(x+3) - (3x+6)}{4(x+3)^{3/2}} = \frac{3x+12}{4(x+3)^{3/2}} = \frac{3(x+4)}{4(x+3)^{3/2}}$$

$A''(x) > 0$ for all $x > -3$, so A is concave upward on $(-3, \infty)$. There is no inflection point.

(d)



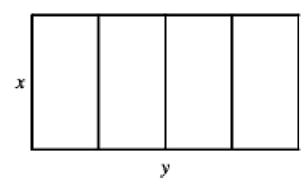
7. (a)



The areas of the three figures are 12,500, 12,500, and 9000 ft^2. There appears to be a maximum area of at least 12,500 ft^2.

(b) Let x denote the length of each of two sides and three dividers.

Let y denote the length of the other two sides.

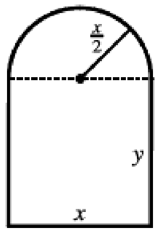


(c) Area $A = \text{length} \times \text{width} = y \cdot x$

(d) Length of fencing $= 750 \quad\Rightarrow\quad 5x + 2y = 750$

(e) $5x + 2y = 750 \quad\Rightarrow\quad y = 375 - \frac{5}{2}x \quad\Rightarrow\quad A(x) = \left(375 - \frac{5}{2}x\right)x = 375x - \frac{5}{2}x^2$

(f) $A'(x) = 375 - 5x = 0 \quad\Rightarrow\quad x = 75$. Since $A''(x) = -5 < 0$ there is an absolute maximum when $x = 75$. Then $y = \frac{375}{2} = 187.5$. The largest area is $75\left(\frac{375}{2}\right) = 14{,}062.5$ ft^2. These values of x and y are between the values in the first and second figures in part (a). Our original estimate was low.

19. 

Perimeter $= 30 \quad \Rightarrow \quad 2y + x + \pi\left(\frac{x}{2}\right) = 30 \quad \Rightarrow$

$y = \frac{1}{2}\left(30 - x - \frac{\pi x}{2}\right) = 15 - \frac{x}{2} - \frac{\pi x}{4}$. The area is the area of the rectangle plus the area of the semicircle, or $xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$, so

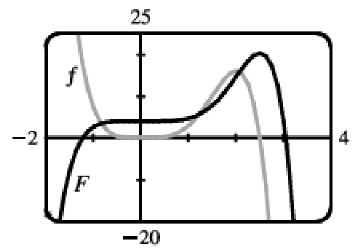
$A(x) = x\left(15 - \frac{x}{2} - \frac{\pi x}{4}\right) + \frac{1}{8}\pi x^2 = 15x - \frac{1}{2}x^2 - \frac{\pi}{8}x^2$.

$A'(x) = 15 - \left(1 + \frac{\pi}{4}\right)x = 0 \quad \Rightarrow \quad x = \dfrac{15}{1 + \pi/4} = \dfrac{60}{4 + \pi}$. $A''(x) = -\left(1 + \frac{\pi}{4}\right) < 0$, so this gives a maximum. The dimensions are $x = \dfrac{60}{4+\pi}$ ft and $y = 15 - \dfrac{30}{4+\pi} - \dfrac{15\pi}{4+\pi} = \dfrac{60 + 15\pi - 30 - 15\pi}{4+\pi} = \dfrac{30}{4+\pi}$ ft, so the height of the rectangle is half the base.

13. $f(x) = 5x^4 - 2x^5 \quad \Rightarrow \quad F(x) = 5 \cdot \dfrac{x^5}{5} - 2 \cdot \dfrac{x^6}{6} + C = x^5 - \frac{1}{3}x^6 + C$.

$F(0) = 4 \quad \Rightarrow \quad 0^5 - \frac{1}{3} \cdot 0^6 + C = 4 \quad \Rightarrow \quad C = 4$, so $F(x) = x^5 - \frac{1}{3}x^6 + 4$.

The graph confirms our answer since $f(x) = 0$ when F has a local maximum, f is positive when F is increasing, and f is negative when F is decreasing.



19. $f'(x) = \sqrt{x}(6 + 5x) = 6x^{1/2} + 5x^{3/2} \quad \Rightarrow \quad f(x) = 4x^{3/2} + 2x^{5/2} + C$.

$f(1) = 6 + C$ and $f(1) = 10 \quad \Rightarrow \quad C = 4$, so $f(x) = 4x^{3/2} + 2x^{5/2} + 4$.

16. For an increasing function, using left endpoints gives us an underestimate and using right endpoints results in an overestimate. We will use M_6 to get an estimate. $\Delta t = \frac{30 - 0}{6} = 5 \text{ s} = \frac{5}{3600} \text{ h} = \frac{1}{720} \text{ h}$.

$$\begin{aligned} M_6 &= \tfrac{1}{720}[v(2.5) + v(7.5) + v(12.5) + v(17.5) + v(22.5) + v(27.5)] \\ &= \tfrac{1}{720}(31.25 + 66 + 88 + 103.5 + 113.75 + 119.25) = \tfrac{1}{720}(521.75) \approx 0.725 \text{ km} \end{aligned}$$

For a very rough check on the above calculation, we can draw a line from $(0, 0)$ to $(30, 120)$ and calculate the area of the triangle: $\frac{1}{2}(30)(120) = 1800$. Divide by 3600 to get 0.5, which is clearly an underestimate, making our midpoint estimate of 0.725 seem reasonable. Of course, answers will vary due to different readings of the graph.

41. $\int_{-2}^{2} f(x)\,dx + \int_{2}^{5} f(x)\,dx - \int_{-2}^{-1} f(x)\,dx = \int_{-2}^{5} f(x)\,dx + \int_{-1}^{-2} f(x)\,dx$ [by Property 5 and reversing limits]

$= \int_{-1}^{5} f(x)\,dx$ [Property 5]

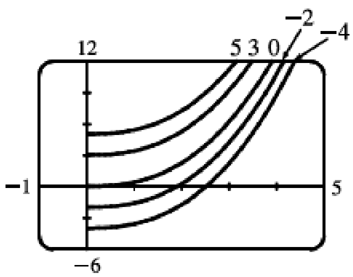
43. $\int_0^9 [2f(x) + 3g(x)]\,dx = 2\int_0^9 f(x)\,dx + 3\int_0^9 g(x)\,dx = 2(37) + 3(16) = 122$

37. $\dfrac{d}{dx}\left[\sqrt{x^2+1} + C\right] = \dfrac{d}{dx}\left[\left(x^2+1\right)^{1/2} + C\right] = \frac{1}{2}\left(x^2+1\right)^{-1/2} \cdot 2x = \dfrac{x}{\sqrt{x^2+1}}$

39. $\int x\sqrt{x}\,dx = \int x^{3/2}\,dx = \frac{2}{5}x^{5/2} + C.$

The members of the family in the figure correspond to

$C = 5, 3, 0, -2,$ and -4.

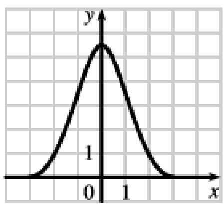


4. (a) $g(-3) = \int_{-3}^{-3} f(t)\,dt = 0$, $g(3) = \int_{-3}^{3} f(t)\,dt = \int_{-3}^{0} f(t)\,dt + \int_{0}^{3} f(t)\,dt = 0$ by symmetry, since the area above the x-axis is the same as the area below the axis.

(b) From the graph, it appears that to the nearest $\frac{1}{2}$, $g(-2) = \int_{-3}^{-2} f(t)\,dt \approx 1$, $g(-1) = \int_{-3}^{-1} f(t)\,dt \approx 3\frac{1}{2}$, and $g(0) = \int_{-3}^{0} f(t)\,dt \approx 5\frac{1}{2}$.

(c) g is increasing on $(-3, 0)$ because as x increases from -3 to 0, we keep adding more area.

(d) g has a maximum value when we start subtracting area; that is, at $x = 0$.

(e) 

(f) The graph of $g'(x)$ is the same as that of $f(x)$, as indicated by FTC1.