

15. For a decreasing function, using left endpoints gives us an overestimate and using right endpoints results in an underestimate. We will use M_6 to get an estimate. $\Delta t = 1$, so

$$M_6 = 1[v(0.5) + v(1.5) + v(2.5) + v(3.5) + v(4.5) + v(5.5)] \\ \approx 55 + 40 + 28 + 18 + 10 + 4 = 155 \text{ ft}$$

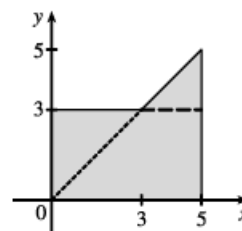
For a very rough check on the above calculation, we can draw a line from $(0, 70)$ to $(6, 0)$ and calculate the area of the triangle: $\frac{1}{2}(70)(6) = 210$. This is clearly an overestimate, so our midpoint estimate of 155 is reasonable.

43. $\int_0^9 [2f(x) + 3g(x)] dx = 2 \int_0^9 f(x) dx + 3 \int_0^9 g(x) dx = 2(37) + 3(16) = 122$

44. If $f(x) = \begin{cases} 3 & \text{for } x < 3 \\ x & \text{for } x \geq 3 \end{cases}$, then $\int_0^5 f(x) dx$ can be interpreted as the area of

the shaded region, which consists of a 5-by-3 rectangle surmounted by an isosceles right triangle whose legs have length 2. Thus,

$$\int_0^5 f(x) dx = 5(3) + \frac{1}{2}(2)(2) = 17.$$



41. $\int (1-t)(2+t^2) dt = \int (2-2t+t^2-t^3) dt = 2t - 2\frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{4} + C = 2t - t^2 + \frac{1}{3}t^3 - \frac{1}{4}t^4 + C$

42. $\int x(1+2x^4) dx = \int (x+2x^5) dx = \frac{x^2}{2} + 2\frac{x^6}{6} + C = \frac{1}{2}x^2 + \frac{1}{3}x^6 + C$

55. (a) Displacement = $\int_0^3 (3t-5) dt = [\frac{3}{2}t^2 - 5t]_0^3 = \frac{27}{2} - 15 = -\frac{3}{2}$ m

(b) Distance traveled = $\int_0^3 |3t-5| dt = \int_0^{5/3} (5-3t) dt + \int_{5/3}^3 (3t-5) dt$
 $= [5t - \frac{3}{2}t^2]_0^{5/3} + [\frac{3}{2}t^2 - 5t]_{5/3}^3 = \frac{25}{3} - \frac{3}{2} \cdot \frac{25}{9} + \frac{27}{2} - 15 - (\frac{3}{2} \cdot \frac{25}{9} - \frac{25}{3}) = \frac{41}{6}$ m

57. (a) $v'(t) = a(t) = t + 4 \Rightarrow v(t) = \frac{1}{2}t^2 + 4t + C \Rightarrow v(0) = C = 5 \Rightarrow v(t) = \frac{1}{2}t^2 + 4t + 5$ m/s

(b) Distance traveled = $\int_0^{10} |v(t)| dt = \int_0^{10} |\frac{1}{2}t^2 + 4t + 5| dt = \int_0^{10} (\frac{1}{2}t^2 + 4t + 5) dt$
 $= [\frac{1}{6}t^3 + 2t^2 + 5t]_0^{10} = \frac{500}{3} + 200 + 50 = 416\frac{2}{3}$ m

3. (a) $g(x) = \int_0^x f(t) dt$.

$$g(0) = \int_0^0 f(t) dt = 0$$

$$g(1) = \int_0^1 f(t) dt = 1 \cdot 2 = 2 \text{ [rectangle]},$$

$$g(2) = \int_0^2 f(t) dt = \int_0^1 f(t) dt + \int_1^2 f(t) dt = g(1) + \int_1^2 f(t) dt \\ = 2 + 1 \cdot 2 + \frac{1}{2} \cdot 1 \cdot 2 = 5 \text{ [rectangle plus triangle]},$$

$$g(3) = \int_0^3 f(t) dt = g(2) + \int_2^3 f(t) dt = 5 + \frac{1}{2} \cdot 1 \cdot 4 = 7,$$

$$g(6) = g(3) + \int_3^6 f(t) dt \text{ [the integral is negative since } f \text{ lies under the } x\text{-axis]} \\ = 7 + \left[-\left(\frac{1}{2} \cdot 2 \cdot 2 + 1 \cdot 2\right) \right] = 7 - 4 = 3$$

(b) g is increasing on $(0, 3)$ because as x increases from 0 to 3, we keep adding more area.

(c) g has a maximum value when we start subtracting area; that is, at $x = 3$.

