Math 520

Area Problem & Distances

§5.1

In this section we will begin to learn the techniques of integration. This is the part of calculus used to find areas, volumes, centers of mass, etc. This section introduces two types of problems—finding areas under curves and finding distances given velocities.

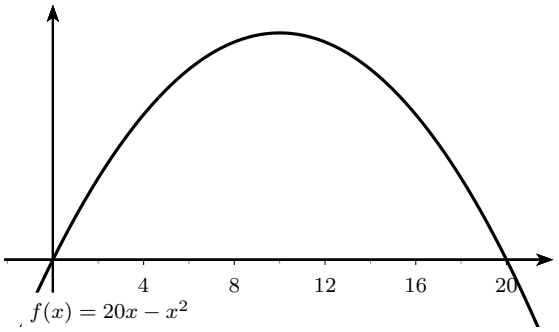
Area Under a Curve

One way to approximate the area under a curve that lies above the $x-$ axis and between the vertical lines $x = a$ and $x = b$ is to cover it with many adjoining rectangles. Then use the area of these rectangles to estimate the area under the curve.

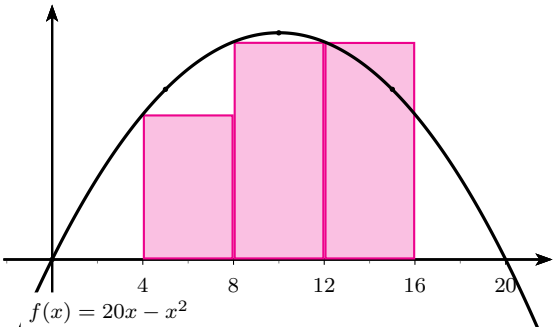
1. Partition the interval $[a, b]$ into n equal subintervals, each with width $\Delta x = \dfrac{b-a}{n}$
2. Select a sample point x_i^* from each subinterval. Usually this choice is the (i) right endpoint, (ii) left endpoint, or (iii) midpoint.
3. Calculate $\displaystyle\sum_{i=1}^{n} f(x_i^*)\Delta x = f(x_1^*)\Delta x + f(x_2^*)\Delta x + f(x_3^*)\Delta x + \cdots + f(x_n^*)\Delta x$

1. Use three rectangles of each type to estimate the area under the curve $y = 20x - x^2$ that lies above the $x-$axis between $x = 4$ and $x = 16$. Use three subintervals of equal width.

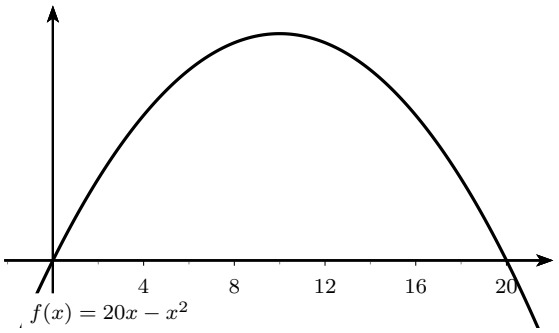
   (a) Using the left endpoints, L_3.



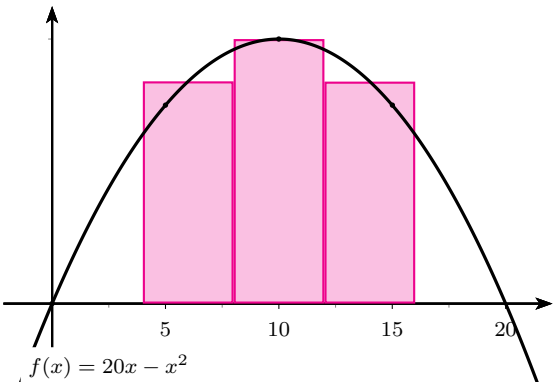
Solution:



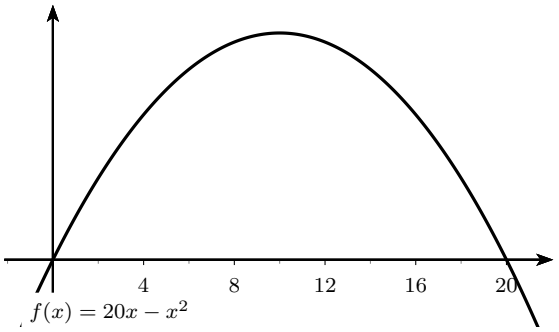
   (b) Using the mid points, M_3.



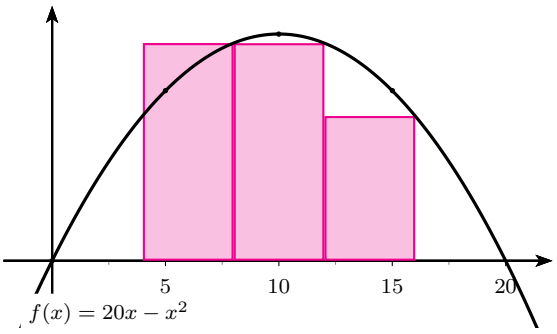
Solution:



(c) Using the right endpoints, R_3



Solution:



2. Consider the area under the function $f(x) = 1 + 4x^2$ that lies above the $x-$axis between $x = 2$ and $x = 8$.

   (a) Partition $[2, 8]$ into 4 subintervals of equal width. Determine the width of each subinterval.

   Solution: $\Delta x = \dfrac{8-2}{4} = 1.5$

   (b) Using the left endpoints as sample points to complete the table below.

i	subinterval	x_i^*	$f(x_i^*)$	$f(x_i^*)\Delta x$
1				
2				
3				
4				

Solution:

i	subinterval	x_i^*	$f(x_i^*)$	$f(x_i^*)\Delta x$
1	$[2, 3.5]$	2	17	25.5
2	$[3.5, 5]$	3.5	50	75
3	$[5, 6.5]$	5	101	151.5
4	$[6.5, 8]$	6.5	170	255

(c) Calculate $\sum_{i=1}^{4} f(x_i^*)\Delta x = f(x_1^*)\Delta x + f(x_2^*)\Delta x + f(x_3^*)\Delta x + f(x_4^*)\Delta x$

Solution: $25.5 + 75 + 151.5 + 225 = 507$