Math 520

Areas between Curves

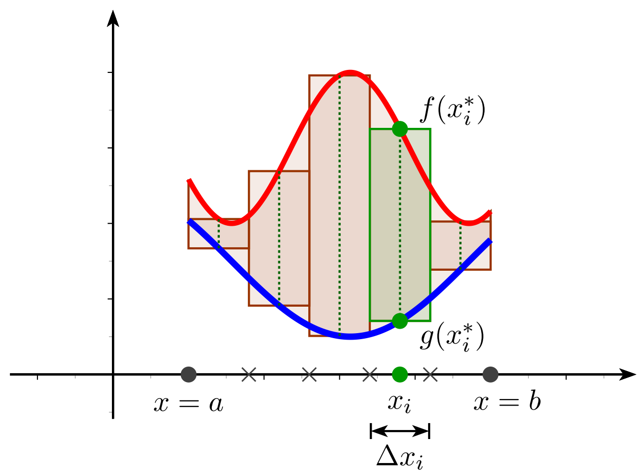
§6.1

In this section we will study how to find the area between curves. It will help to remember that each application arises naturally as limit of Riemann sums.

Area between Curves

The area A of the region bounded by the graphs of continuous functions $y = f(x)$ and $y = g(x)$ and $x = a$, $x = b$ with $f(x) \geq g(x)$ on $[a, b]$ may be approximated by a Riemann sum.

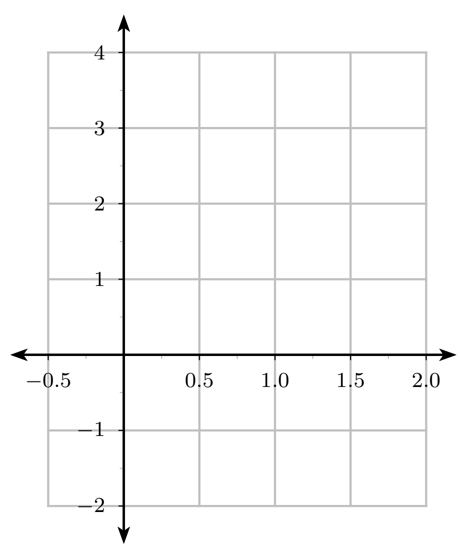
$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left[f(x_i^*) - g(x_i^*)\right]\Delta x_i = \int_a^b [(f(x) - g(x)]\,dx = \int_a^b y_{top} - y_{bottom}\,dx.$$



1. (a) Using the axis below, sketch the graphs of $y = x^3$ and $y = 3 - x$.

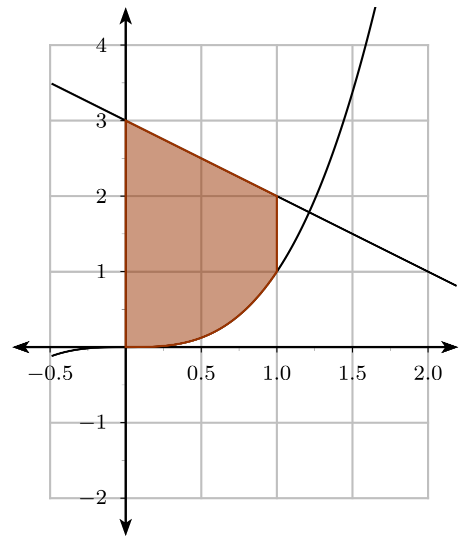
   (b) Shade the region bounded by $y = x^3$, $y = 3 - x$, $x = 0$, and $x = 1$.

   (c) Write an expression involving a definite integral that represents the area of the region shaded in (b).



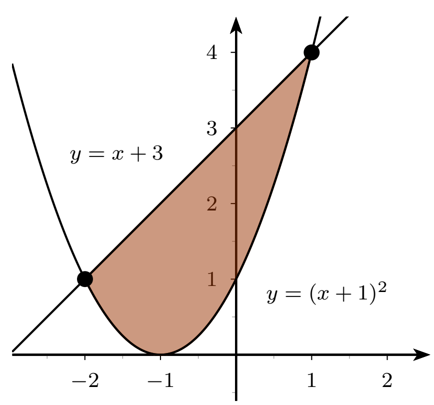
Solution:

Area of the shaded region is $\int_0^1 (3 - x) - x^3\,dx$

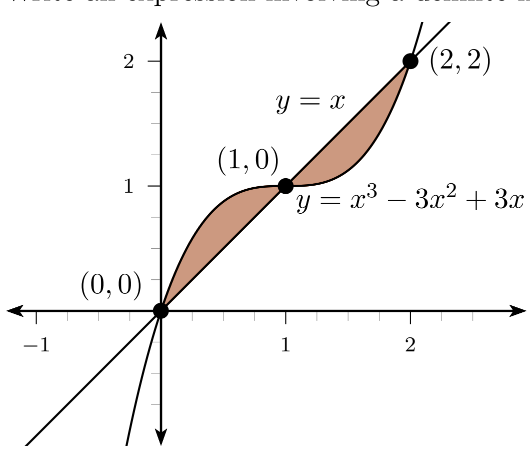


2. Write an expression involving a definite integral for the area of the region bounded by $y = (x+1)^2$ and $y = x + 3$.

Solution: The area of the region is $\int_{-2}^{1} [(x+3) - (x+1)^2]\,dx$.



3. Write an expression involving a definite integral for the area of the region below.

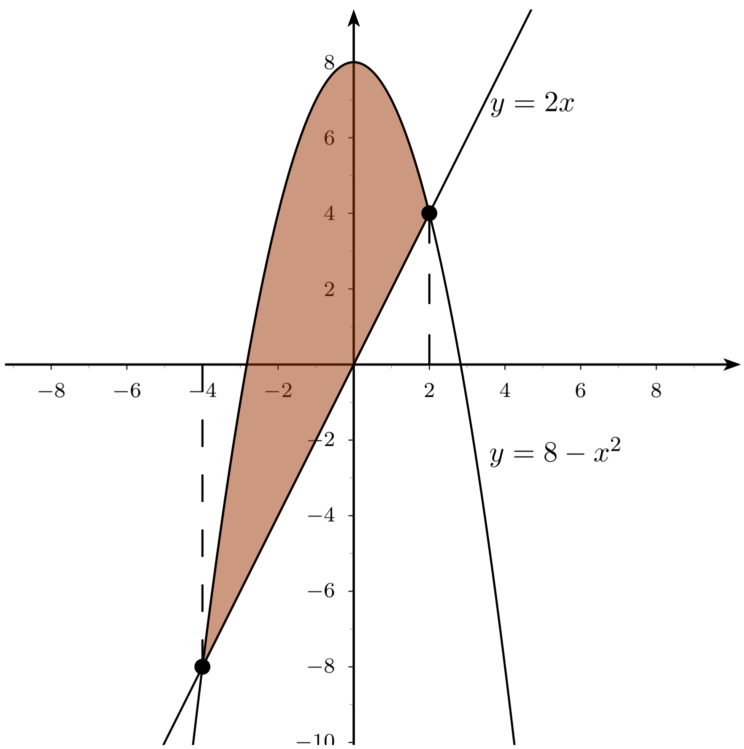


Solution: $\int_0^1 [(x^3 - 3x^2 + 3x) - x]\, dx + \int_1^2 [x - (x^3 - 3x^2 + 3x)]\, dx$

4. Write an expression involving a definite integral for the area of the regions bounded by $y = 2x$ and $y = 8 - x^2$.

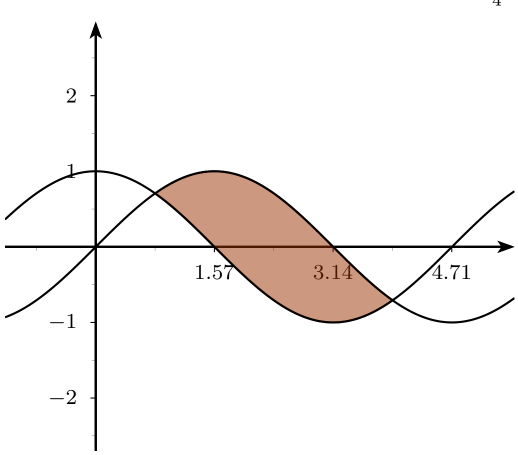
Solution:

The shaded are is $\int_{-4}^{2} (8 - x^2 - 2x)\, dx$



5. Write an expression involving a definite integral for the area of the regions bounded by $y = \cos x$ and $y = \sin x$ on $[0, 2\pi]$.

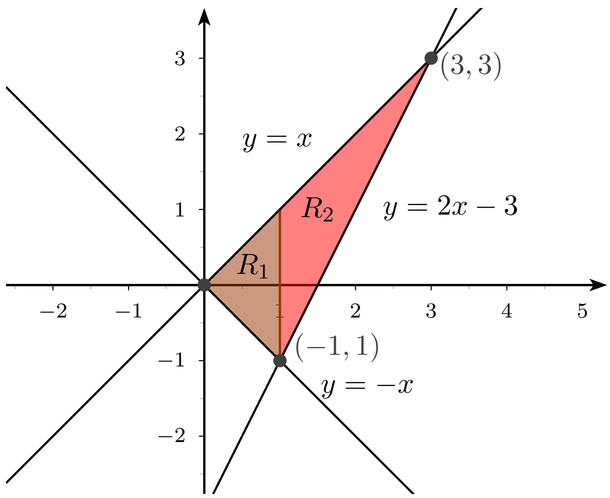
Solution: The area of the region is $\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \cos x - \sin x\, dx$



6. Write an expression involving a definite integral for the area of the regions bounded by $y = x$, $y = -x$, and $y = 2x - 3$.

Solution: The area of the shaded region is

$$R_1 + R_2 = \int_0^1 [x - (-x)]\, dx + \int_1^3 [x - (2x - 3)]\, dx = \int_0^1 2x\, dx + \int_1^3 (3 - x)\, dx.$$



7. Write an expression involving a definite integral for the area of the regions bounded by $f(x) = 3x^3 - x^2 - 10x$ and $g(x) = -x^2 + 2x$.

Solution: The area of the shaded region is

$$\int_{-2}^{0} (f(x) - g(x))\, dx + \int_0^{-2} (g(x) - f(x))\, dx = \int_{-2}^{0} (3x^3 - 12x)\, dx + \int_0^2 (-3x^3 + 12x)\, dx.$$