Math 520

Antiderivatives

§4.9

In this section we will begin to learn how to "undo" derivatives; that is, given a derivative, recover the function from where it came. We will see that there is not just one antiderivative function, but rather a whole collection of functions, each of which differ by a constant from one another

Antiderivatives

A function F is called an **antiderivative** of f on an interval I iff

$$F'(x) = f(x)$$

for all $x \in I$ and the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

Function	Form of All Antiderivatives	Function	Form of All Antiderivatives
$cf(x)$	$cF(x) + C$	$\sin x$	$-\cos x + C$
$f(x) + g(x)$	$F(x) + G(x) + C$	$\sec^2 x$	$\tan x + C$
x^n (except $n = -1$)	$\frac{x^{n+1}}{n+1} + C$	$\sec x \tan x$	$\sec x + C$
$\frac{1}{x}$	$\ln\lvert x\rvert + C$	$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x + C$
$\cos x$	$\sin x + C$	$\frac{1}{1+x^2}$	$\tan^{-1} x + C$

1. Find all antiderivatives of:
   (a) $f(x) = x^7$

   Solution: $\frac{x^8}{8} + C$

   (b) $f(x) = \cos x - \sec^2 x$

   Solution: $\sin x - \tan x + C$

   (c) $f(x) = x + x^{-2}$

   Solution: $\frac{x^2}{2} - \frac{1}{x} + C$

   (d) $f(x) = x - e^x$

   Solution: $\frac{x^2}{2} + e^x + C$

   (e) $f(x) = \frac{2}{\sqrt{x}}$

Solution: $4x^{\frac{1}{2}} + C$

(f) $f(x) = \dfrac{4}{\sqrt{1-x^2}}$

Solution: $4\sin^{-1} x + C$

2. Find $f(x)$ where $f(2) = 3$ and $f'(x) = 4x + 5$.

Solution:

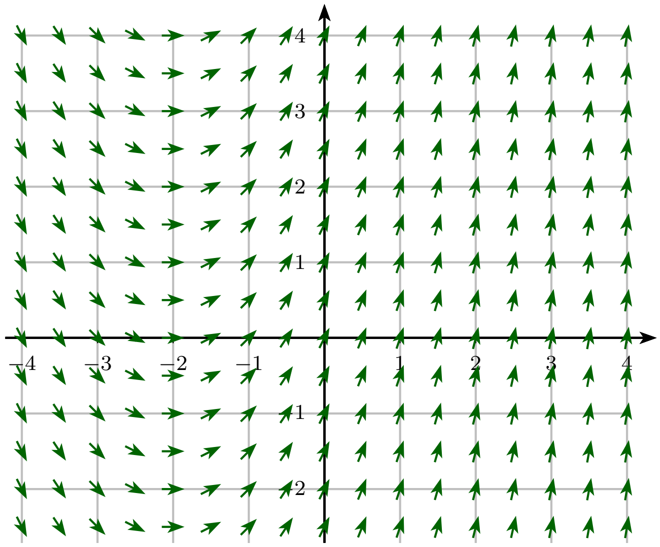
$$\begin{aligned} f(x) &= 2x^2 + 5x + C \\ f(2) &= 2(2)^2 + 5(2) + C = 18 + C \\ 3 &= 18 + C \\ C &= -15 \\ f(x) &= 2x^2 + 5x - 15 \end{aligned}$$

3. A particle moves along a line with velocity $v = 3t + 7$. If the particle is at 4 on the line when $t = 1$, find the position function $s(t)$.

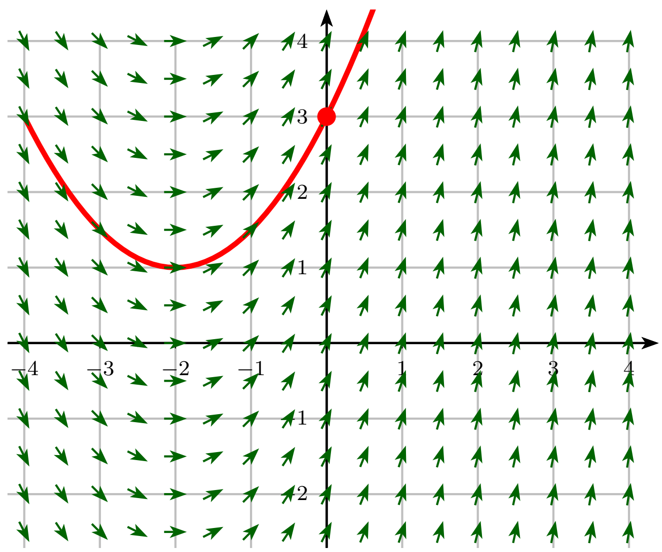
Solution:

$$\begin{aligned} s(t) &= \frac{3}{2}t^2 + 7t + C \\ s(1) &= \frac{3}{2}(1)^2 + 7(1) + C = \frac{17}{2} + C \\ 4 &= \frac{17}{2} + C \\ C &= -\frac{9}{2} \\ s(t) &= \frac{3}{2}t^2 + 7t - \frac{9}{2} \end{aligned}$$

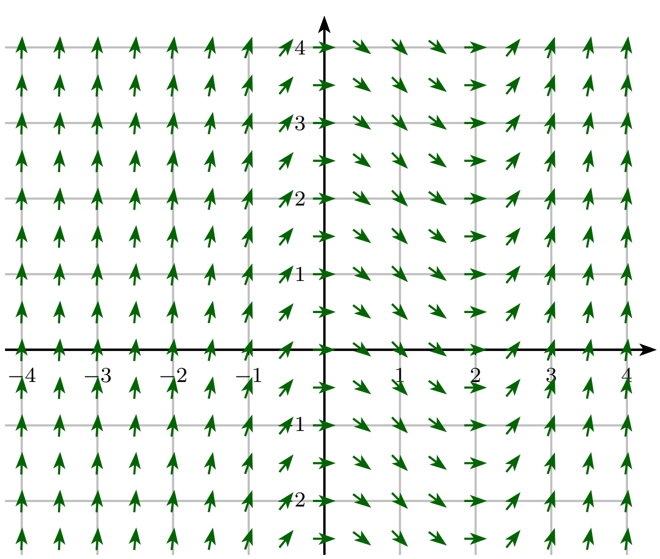
4. The direction field is given for a function. Use it to draw the antiderivative F that satisfies $F(0) = 3$.



Solution:



5. The direction field is given for a function. Use it to draw the antiderivative F that satisfies $F(0) = 3$.



Solution: