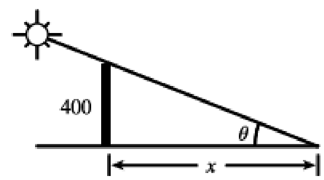
1. $f(x) = 10 + 27x - x^3$, $0 \le x \le 4$. $f'(x) = 27 - 3x^2 = -3(x^2 - 9) = -3(x+3)(x-3) = 0$ only when $x = 3$ (since -3 is not in the domain). $f'(x) > 0$ for $x < 3$ and $f'(x) < 0$ for $x > 3$, so $f(3) = 64$ is a local maximum value. Checking the endpoints, we find $f(0) = 10$ and $f(4) = 54$. Thus, $f(0) = 10$ is the absolute minimum value and $f(3) = 64$ is the absolute maximum value.

2. $f(x) = x - \sqrt{x}$, $0 \le x \le 4$. $f'(x) = 1 - 1/(2\sqrt{x}) = 0 \quad \Leftrightarrow \quad 2\sqrt{x} = 1 \quad \Rightarrow \quad x = \frac{1}{4}$. $f'(x)$ does not exist $\quad \Leftrightarrow$ $x = 0$. $f'(x) < 0$ for $0 < x < \frac{1}{4}$ and $f'(x) > 0$ for $\frac{1}{4} < x < 4$, so $f\left(\frac{1}{4}\right) = -\frac{1}{4}$ is a local and absolute minimum value. $f(0) = 0$ and $f(4) = 2$, so $f(4) = 2$ is the absolute maximum value.

3. $f(x) = \dfrac{x}{x^2 + x + 1}$, $-2 \le x \le 0$. $f'(x) = \dfrac{(x^2 + x + 1)(1) - x(2x + 1)}{(x^2 + x + 1)^2} = \dfrac{1 - x^2}{(x^2 + x + 1)^2} = 0 \quad \Leftrightarrow \quad x = -1$ (since 1 is not in the domain). $f'(x) < 0$ for $-2 < x < -1$ and $f'(x) > 0$ for $-1 < x < 0$, so $f(-1) = -1$ is a local and absolute minimum value. $f(-2) = -\frac{2}{3}$ and $f(0) = 0$, so $f(0) = 0$ is an absolute maximum value.

4. $f(x) = \dfrac{\ln x}{x^2}$, $[1, 3]$. $f'(x) = \dfrac{x^2 \cdot \frac{1}{x} - (\ln x)(2x)}{(x^2)^2} = \dfrac{x - 2x \ln x}{x^4} = \dfrac{1 - 2\ln x}{x^3} = 0 \quad \Leftrightarrow \quad \ln x = \frac{1}{2} \quad \Leftrightarrow$ $x = e^{1/2} = \sqrt{e} \approx 1.65$. $f'(x) > 0$ for $x < \sqrt{e}$ and $f'(x) < 0$ for $x > \sqrt{e}$, so f is increasing on $(1, \sqrt{e})$ and decreasing on $(\sqrt{e}, 3)$. Hence, $f(\sqrt{e}) = \dfrac{1}{2e}$ is a local maximum value. $f(1) = 0$ and $f(3) = \dfrac{\ln 3}{9} \approx 0.12$. Since $\dfrac{1}{2e} \approx 0.18$, $f(\sqrt{e}) = \dfrac{1}{2e}$ is the absolute maximum value and $f(1) = 0$ is the absolute minimum value.

33. We are given $d\theta/dt = -0.25$ rad/h. $\tan\theta = 400/x \quad \Rightarrow$ $x = 400\cot\theta \quad \Rightarrow \quad \dfrac{dx}{dt} = -400\csc^2\theta\,\dfrac{d\theta}{dt}$. When $\theta = \frac{\pi}{6}$, $\dfrac{dx}{dt} = -400(2)^2(-0.25) = 400$ ft/h.



35. Given $dh/dt = 5$ and $dx/dt = 15$, find dz/dt. $z^2 = x^2 + h^2 \quad \Rightarrow$ $2z\,\dfrac{dz}{dt} = 2x\,\dfrac{dx}{dt} + 2h\,\dfrac{dh}{dt} \quad \Rightarrow \quad \dfrac{dz}{dt} = \dfrac{1}{z}(15x + 5h)$. When $t = 3$, $h = 45 + 3(5) = 60$ and $x = 15(3) = 45 \quad \Rightarrow \quad z = \sqrt{45^2 + 60^2} = 75$, so $\dfrac{dz}{dt} = \frac{1}{75}\,[15(45) + 5(60)] = 13$ ft/s.