

21. This limit has the form $\frac{0}{0}$. $\lim_{x \rightarrow 1} \frac{1-x+\ln x}{1+\cos \pi x} \stackrel{H}{=} \lim_{x \rightarrow 1} \frac{-1+1/x}{-\pi \sin \pi x} \stackrel{H}{=} \lim_{x \rightarrow 1} \frac{-1/x^2}{-\pi^2 \cos \pi x} = \frac{-1}{-\pi^2(-1)} = -\frac{1}{\pi^2}$

26. $\lim_{x \rightarrow -\infty} x^2 e^x = \lim_{x \rightarrow -\infty} \frac{x^2}{e^{-x}} \stackrel{H}{=} \lim_{x \rightarrow -\infty} \frac{2x}{-e^{-x}} \stackrel{H}{=} \lim_{x \rightarrow -\infty} \frac{2}{e^{-x}} = \lim_{x \rightarrow -\infty} 2e^x = 0$

27. This limit has the form $\infty \cdot 0$. We'll change it to the form $\frac{0}{0}$.

$$\lim_{x \rightarrow 0} \cot 2x \sin 6x = \lim_{x \rightarrow 0} \frac{\sin 6x}{\tan 2x} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{6 \cos 6x}{2 \sec^2 2x} = \frac{6(1)}{2(1)^2} = 3$$

29. This limit has the form $\infty \cdot 0$. $\lim_{x \rightarrow \infty} x^3 e^{-x^2} = \lim_{x \rightarrow \infty} \frac{x^3}{e^{x^2}} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{3x^2}{2xe^{x^2}} = \lim_{x \rightarrow \infty} \frac{3x}{2e^{x^2}} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{3}{4xe^{x^2}} = 0$

33. The limit has the form $\infty - \infty$ and we will change the form to a product by factoring out x .

$$\lim_{x \rightarrow \infty} (x - \ln x) = \lim_{x \rightarrow \infty} x \left(1 - \frac{\ln x}{x} \right) = \infty \text{ since } \lim_{x \rightarrow \infty} \frac{\ln x}{x} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{1/x}{1} = 0.$$

35. $y = x^{x^2} \Rightarrow \ln y = x^2 \ln x$, so $\lim_{x \rightarrow 0^+} \ln y = \lim_{x \rightarrow 0^+} x^2 \ln x = \lim_{x \rightarrow 0^+} \frac{\ln x}{1/x^2} \stackrel{H}{=} \lim_{x \rightarrow 0^+} \frac{1/x}{-2/x^3} = \lim_{x \rightarrow 0^+} \left(-\frac{1}{2} x^2 \right) = 0 \Rightarrow$

$$\lim_{x \rightarrow 0^+} x^{x^2} = \lim_{x \rightarrow 0^+} e^{\ln y} = e^0 = 1.$$