

1. (a) $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ is an indeterminate form of type $\frac{0}{0}$.
- (b) $\lim_{x \rightarrow a} \frac{f(x)}{p(x)} = 0$ because the numerator approaches 0 while the denominator becomes large.
- (c) $\lim_{x \rightarrow a} \frac{h(x)}{p(x)} = 0$ because the numerator approaches a finite number while the denominator becomes large.
- (d) If $\lim_{x \rightarrow a} p(x) = \infty$ and $f(x) \rightarrow 0$ through positive values, then $\lim_{x \rightarrow a} \frac{p(x)}{f(x)} = \infty$. [For example, take $a = 0$, $p(x) = 1/x^2$, and $f(x) = x^2$.] If $f(x) \rightarrow 0$ through negative values, then $\lim_{x \rightarrow a} \frac{p(x)}{f(x)} = -\infty$. [For example, take $a = 0$, $p(x) = 1/x^2$, and $f(x) = -x^2$.] If $f(x) \rightarrow 0$ through both positive and negative values, then the limit might not exist. [For example, take $a = 0$, $p(x) = 1/x^2$, and $f(x) = x$.]
- (e) $\lim_{x \rightarrow a} \frac{p(x)}{q(x)}$ is an indeterminate form of type $\frac{\infty}{\infty}$.

5. This limit has the form $\frac{0}{0}$. We can simply factor the numerator to evaluate this limit.

$$\lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1} = \lim_{x \rightarrow -1} \frac{(x + 1)(x - 1)}{x + 1} = \lim_{x \rightarrow -1} (x - 1) = -2$$

7. This limit has the form $\frac{0}{0}$. $\lim_{x \rightarrow (\pi/2)^+} \frac{\cos x}{1 - \sin x} \stackrel{H}{=} \lim_{x \rightarrow (\pi/2)^+} \frac{-\sin x}{-\cos x} = \lim_{x \rightarrow (\pi/2)^+} \tan x = -\infty$.

9. This limit has the form $\frac{0}{0}$. $\lim_{t \rightarrow 0} \frac{e^t - 1}{t^3} \stackrel{H}{=} \lim_{t \rightarrow 0} \frac{e^t}{3t^2} = \infty$ since $e^t \rightarrow 1$ and $3t^2 \rightarrow 0^+$ as $t \rightarrow 0$.

11. This limit has the form $\frac{0}{0}$. $\lim_{x \rightarrow 0} \frac{\tan px}{\tan qx} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{p \sec^2 px}{q \sec^2 qx} = \frac{p(1)^2}{q(1)^2} = \frac{p}{q}$

13. $\lim_{x \rightarrow 0^+} [(\ln x)/x] = -\infty$ since $\ln x \rightarrow -\infty$ as $x \rightarrow 0^+$ and dividing by small values of x just increases the magnitude of the quotient $(\ln x)/x$. L'Hospital's Rule does not apply.

15. This limit has the form $\frac{0}{0}$. $\lim_{t \rightarrow 0} \frac{5^t - 3^t}{t} \stackrel{H}{=} \lim_{t \rightarrow 0} \frac{5^t \ln 5 - 3^t \ln 3}{1} = \ln 5 - \ln 3 = \ln \frac{5}{3}$

17. This limit has the form $\frac{0}{0}$. $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{e^x - 1}{2x} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{e^x}{2} = \frac{1}{2}$

19. This limit has the form $\frac{\infty}{\infty}$. $\lim_{x \rightarrow \infty} \frac{x}{\ln(1 + 2e^x)} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{1}{\frac{1}{1 + 2e^x} \cdot 2e^x} = \lim_{x \rightarrow \infty} \frac{1 + 2e^x}{2e^x} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{2e^x}{2e^x} = 1$