7. (a) $f(x) = x^3 - 12x + 1 \quad \Rightarrow \quad f'(x) = 3x^2 - 12 = 3(x+2)(x-2)$.
We don't need to include "3" in the chart to determine the sign of $f'(x)$.

Interval	$x+2$	$x-2$	$f'(x)$	f
$x < -2$	$-$	$-$	$+$	increasing on $(-\infty, -2)$
$-2 < x < 2$	$+$	$-$	$-$	decreasing on $(-2, 2)$
$x > 2$	$+$	$+$	$+$	increasing on $(2, \infty)$

So f is increasing on $(-\infty, -2)$ and $(2, \infty)$ and f is decreasing on $(-2, 2)$.

(b) f changes from increasing to decreasing at $x = -2$ and from decreasing to increasing at $x = 2$. Thus, $f(-2) = 17$ is a local maximum value and $f(2) = -15$ is a local minimum value.

(c) $f''(x) = 6x$. $f''(x) > 0 \quad \Leftrightarrow \quad x > 0$ and $f''(x) < 0 \quad \Leftrightarrow \quad x < 0$. Thus, f is concave upward on $(0, \infty)$ and concave downward on $(-\infty, 0)$. There is an inflection point where the concavity changes, at $(0, f(0)) = (0, 1)$.

9. (a) $f(x) = x - 2\sin x$ on $(0, 3\pi) \quad \Rightarrow \quad f'(x) = 1 - 2\cos x$. $\quad f'(x) > 0 \quad \Leftrightarrow \quad 1 - 2\cos x > 0 \quad \Leftrightarrow \quad \cos x < \frac{1}{2} \quad \Leftrightarrow$ $\frac{\pi}{3} < x < \frac{5\pi}{3}$ or $\frac{7\pi}{3} < x < 3\pi$. $f'(x) < 0 \quad \Leftrightarrow \quad \cos x > \frac{1}{2} \quad \Leftrightarrow \quad 0 < x < \frac{\pi}{3}$ or $\frac{5\pi}{3} < x < \frac{7\pi}{3}$. So f is increasing on $\left(\frac{\pi}{3}, \frac{5\pi}{3}\right)$ and $\left(\frac{7\pi}{3}, 3\pi\right)$, and f is decreasing on $\left(0, \frac{\pi}{3}\right)$ and $\left(\frac{5\pi}{3}, \frac{7\pi}{3}\right)$.

(b) f changes from increasing to decreasing at $x = \frac{5\pi}{3}$, and from decreasing to increasing at $x = \frac{\pi}{3}$ and at $x = \frac{7\pi}{3}$. Thus, $f\left(\frac{5\pi}{3}\right) = \frac{5\pi}{3} + \sqrt{3} \approx 6.97$ is a local maximum value and $f\left(\frac{\pi}{3}\right) = \frac{\pi}{3} - \sqrt{3} \approx -0.68$ and $f\left(\frac{7\pi}{3}\right) = \frac{7\pi}{3} - \sqrt{3} \approx 5.60$ are local minimum values.

(c) $f''(x) = 2\sin x > 0 \quad \Leftrightarrow \quad 0 < x < \pi$ and $2\pi < x < 3\pi$, $f''(x) < 0 \quad \Leftrightarrow \quad \pi < x < 2\pi$. Thus, f is concave upward on $(0, \pi)$ and $(2\pi, 3\pi)$, and f is concave downward on $(\pi, 2\pi)$. There are inflection points at (π, π) and $(2\pi, 2\pi)$.

11. (a) $y = f(x) = xe^x \quad \Rightarrow \quad f'(x) = xe^x + e^x = e^x(x+1)$. So $f'(x) > 0 \quad \Leftrightarrow \quad x + 1 > 0 \quad \Leftrightarrow \quad x > -1$. Thus, f is increasing on $(-1, \infty)$ and decreasing on $(-\infty, -1)$.

(b) f changes from decreasing to increasing at its only critical number, $x = -1$. Thus, $f(-1) = -e^{-1}$ is a local minimum value.

(c) $f'(x) = e^x(x+1) \quad \Rightarrow \quad f''(x) = e^x(1) + (x+1)e^x = e^x(x+2)$. So $f''(x) > 0 \quad \Leftrightarrow \quad x + 2 > 0 \quad \Leftrightarrow \quad x > -2$. Thus, f is concave upward on $(-2, \infty)$ and concave downward on $(-\infty, -2)$. Since the concavity changes direction at $x = -2$, the point $\left(-2, -2e^{-2}\right)$ is an inflection point.

27. (a) $f(\theta) = 2\cos\theta - \cos 2\theta,\ \ 0 \le \theta \le 2\pi.$

$f'(\theta) = -2\sin\theta + 2\sin 2\theta = -2\sin\theta + 2(2\sin\theta\cos\theta) = 2\sin\theta\,(2\cos\theta - 1).$

Interval	$\sin\theta$	$2\cos\theta - 1$	$f'(\theta)$	f
$0 < \theta < \frac{\pi}{3}$	$+$	$+$	$+$	increasing on $(0, \frac{\pi}{3})$
$\frac{\pi}{3} < \theta < \pi$	$+$	$-$	$-$	decreasing on $(\frac{\pi}{3}, \pi)$
$\pi < \theta < \frac{5\pi}{3}$	$-$	$-$	$+$	increasing on $(\pi, \frac{5\pi}{3})$
$\frac{5\pi}{3} < \theta < 2\pi$	$-$	$+$	$-$	decreasing on $(\frac{5\pi}{3}, 2\pi)$

(b) $f\left(\frac{\pi}{3}\right) = \frac{3}{2}$ and $f\left(\frac{5\pi}{3}\right) = \frac{3}{2}$ are local maximum values and $f(\pi) = -3$ is a local minimum value.

(c) $f'(\theta) = -2\sin\theta + 2\sin 2\theta \quad \Rightarrow$

$$f''(\theta) = -2\cos\theta + 4\cos 2\theta = -2\cos\theta + 4(2\cos^2\theta - 1) = 2(4\cos^2\theta - \cos\theta - 2)$$

$$f''(\theta) = 0 \quad \Leftrightarrow \quad \cos\theta = \frac{1 \pm \sqrt{33}}{8} \quad \Leftrightarrow \quad \theta = \cos^{-1}\left(\frac{1 \pm \sqrt{33}}{8}\right)$$

$$\Leftrightarrow \quad \theta = \cos^{-1}\left(\frac{1 + \sqrt{33}}{8}\right) \approx 0.5678,$$

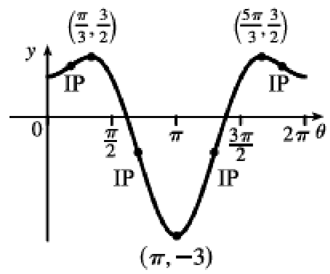
$2\pi - \cos^{-1}\left(\frac{1 + \sqrt{33}}{8}\right) \approx 5.7154$, $\cos^{-1}\left(\frac{1 - \sqrt{33}}{8}\right) \approx 2.2057$, or $2\pi - \cos^{-1}\left(\frac{1 - \sqrt{33}}{8}\right) \approx 4.0775$. Denote these four values of θ by θ_1, θ_4, θ_2, and θ_3, respectively. Then f is CU on $(0, \theta_1)$, CD on (θ_1, θ_2), CU on (θ_2, θ_3), CD on (θ_3, θ_4), and CU on $(\theta_4, 2\pi)$. To find the *exact* y-coordinate for $\theta = \theta_1$, we have

$$f(\theta_1) = 2\cos\theta_1 - \cos 2\theta_1 = 2\cos\theta_1 - \left(2\cos^2\theta_1 - 1\right) = 2\left(\frac{1 + \sqrt{33}}{8}\right) - 2\left(\frac{1 + \sqrt{33}}{8}\right)^2 + 1$$

$$= \tfrac{1}{4} + \tfrac{1}{4}\sqrt{33} - \tfrac{1}{32} - \tfrac{1}{16}\sqrt{33} - \tfrac{33}{32} + 1 = \tfrac{3}{16} + \tfrac{3}{16}\sqrt{33} = \tfrac{3}{16}\left(1 + \sqrt{33}\right) = y_1 \approx 1.26.$$

Similarly, $f(\theta_2) = \frac{3}{16}\left(1 - \sqrt{33}\right) = y_2 \approx -0.89$. So f has inflection points at (θ_1, y_1), (θ_2, y_2), (θ_3, y_2), and (θ_4, y_1).

(d)



29. $f(x) = \dfrac{x^2}{x^2-1} = \dfrac{x^2}{(x+1)(x-1)}$ has domain $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$.

(a) $\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} \dfrac{x^2/x^2}{(x^2-1)/x^2} = \lim\limits_{x\to\pm\infty} \dfrac{1}{1-1/x^2} = \dfrac{1}{1-0} = 1$, so $y=1$ is a HA.

$\lim\limits_{x\to -1^-} \dfrac{x^2}{x^2-1} = \infty$ since $x^2 \to 1$ and $(x^2-1)\to 0^+$ as $x\to -1^-$, so $x=-1$ is a VA.

$\lim\limits_{x\to 1^+} \dfrac{x^2}{x^2-1} = \infty$ since $x^2 \to 1$ and $(x^2-1)\to 0^+$ as $x\to 1^+$, so $x=1$ is a VA.

(b) $f(x) = \dfrac{x^2}{x^2-1} \quad\Rightarrow\quad f'(x) = \dfrac{(x^2-1)(2x)-x^2(2x)}{(x^2-1)^2} = \dfrac{2x[(x^2-1)-x^2]}{(x^2-1)^2} = \dfrac{-2x}{(x^2-1)^2}$. Since $(x^2-1)^2$ is positive for all x in the domain of f, the sign of the derivative is determined by the sign of $-2x$. Thus, $f'(x)>0$ if $x<0$ $(x\neq -1)$ and $f'(x)<0$ if $x>0$ $(x\neq 1)$. So f is increasing on $(-\infty,-1)$ and $(-1,0)$, and f is decreasing on $(0,1)$ and $(1,\infty)$.

(c) $f'(x)=0 \quad\Rightarrow\quad x=0$ and $f(0)=0$ is a local maximum value.

(d) $$f''(x) = \frac{(x^2-1)^2(-2)-(-2x)\cdot 2(x^2-1)(2x)}{[(x^2-1)^2]^2}$$
$$= \frac{2(x^2-1)[-(x^2-1)+4x^2]}{(x^2-1)^4} = \frac{2(3x^2+1)}{(x^2-1)^3}.$$

The sign of $f''(x)$ is determined by the denominator; that is, $f''(x)>0$ if $|x|>1$ and $f''(x)<0$ if $|x|<1$. Thus, f is CU on $(-\infty,-1)$ and $(1,\infty)$, and f is CD on $(-1,1)$. There are no inflection points.

(e)