Math 575

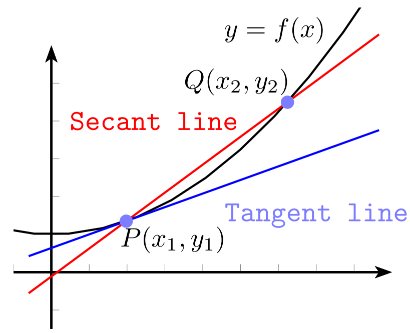
Tangent and Velocity Problems

§2.1

A **secant line** at $P(a, b)$ for the graph of $y = f(x)$ is a line joining P and another point Q also on the graph. If Q has coordinates (c, d) then the slope of the secant line is

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

A **tangent line** to a graph of a function touches the graph at a point much like a tangent line to a circle.



To find the slope of the tangent line at P, we successively select points Q closer and closer to P. As we do this, the slopes of the secant lines become a better and better estimate of the slope of the tangent line at P. Finally, our guess for the slope of the tangent line is the value that the slopes of the secant lines seem to be approaching as points Q get closer and closer to P.

Once we had determined the slope m of the tangent line to the curve, then using the point P as a point on the line, the equation of the tangent line is

$$y - y_1 = m(x - x_1).$$

1. Let point P be the point $(3, 10)$ on the graph of $y = f(x)$ and let Q be the differing points given in the table. Find the slopes of each secant line PQ.

Q	Slope of PQ
$(6, 18)$	
$(5, 15)$	
$(4, 12.3)$	
$(3.5, 11.1)$	
$(3.1, 10.21)$	

2. Let $P(1, 5)$ be a point on the graph of $f(x) = 6x - x^2$. Let $Q(x, 6x - x^2)$ be on the graph. Find the slope of the secant line PQ for each given value for Q.

x	$f(x)$	Slope of PQ
3		
2		
1.5		
1.01		

3. Use your answer to question 2 to guess the slope of the tangent line to $f(x)$ at P.

4. What is the equation of the tangent line at P in question 2?

If $f(x)$ is interpreted as the distance an object is located from the origin along an $x-$axis at time x, then:

- the slope of the secant line through P and Q is the **average velocity** from P to Q.
- the slope of the tangent line through P is the **instantaneous velocity** at P.

When using the distance function:

to find the average velocity, calculate the slope of a secant line, and calculating instantaneous velocity is performed in the same manner as calculating the slope of a tangent line.

5. The distance, in feet, of a ball thrown from the origin is given by the equation

$$s(t) = t^2 + 3t$$

after t seconds.

   (a) What is the average velocity over the time interval $t = 2$ to $t = 4$?

   (b) What is the instantaneous velocity at $t = 2$?

6. The distance that a runner is from the starting line is given in the following table:

t (seconds)	0	1	2	3	4
d (meters)	0	4	9	16	24

   (a) Find the average velocity over the time intervals $[1,4]$, $[1,3]$, $[1,2]$, and $[0,1]$.

   (b) Estimate the instantaneous velocity at $t = 1$.