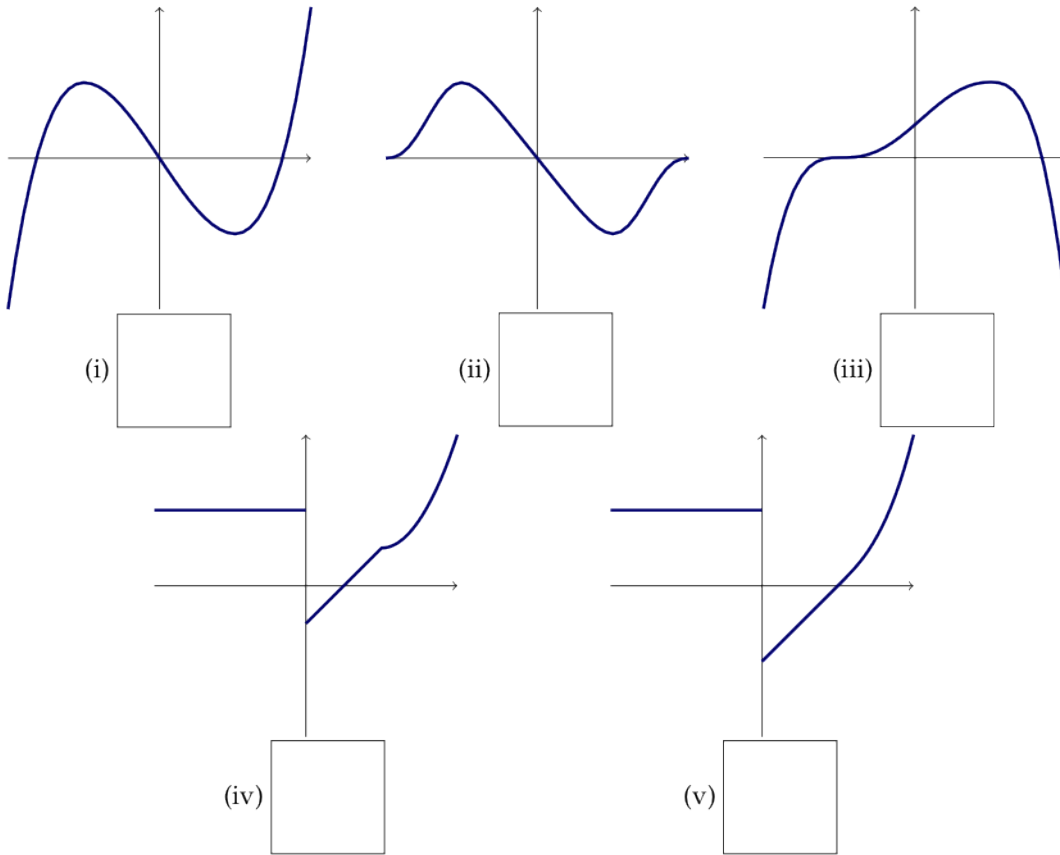


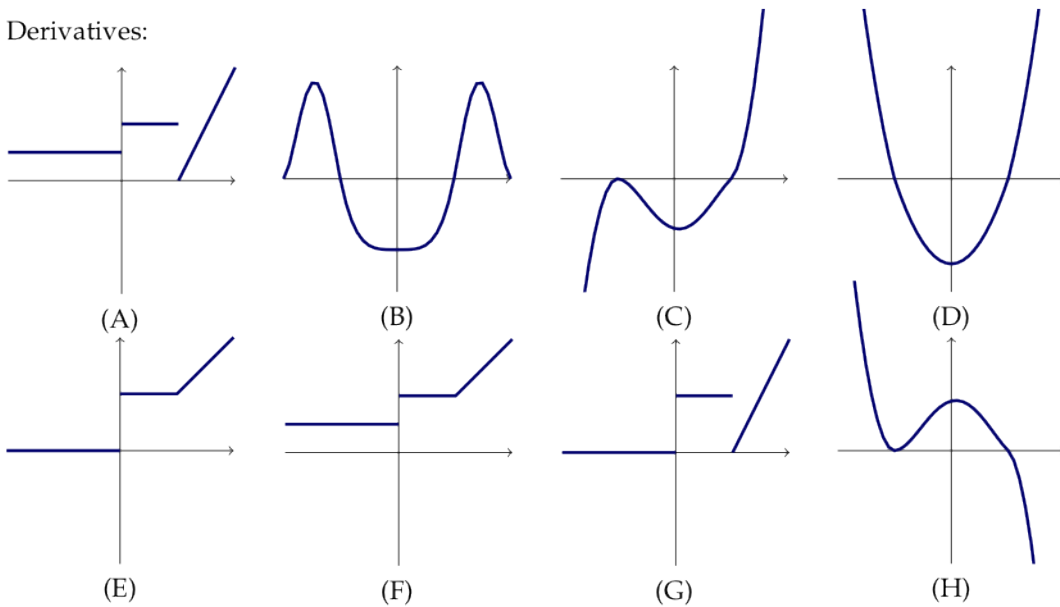
# Math 510

## Exam Review 3

1. Match the graphs of each of the five functions below with the graphs of their derivatives.



Derivatives:



**Answer:**

(i) D    (ii) B    (iii) H    (iv) G    (v) F

2. Determine the following derivatives.

- (a)  $\frac{d}{dx}x^3 + 3x^2 - 4$   
 (b)  $\frac{d^2}{dx^2}x^3 + 3x^2 - 4$   
 (c)  $\frac{d}{dx}3e^x$   
 (d)  $\frac{d}{dx}3e^x + e^3 + x^e$   
 (e)  $\frac{d}{dx}\pi^3$

**Answer:**

- (a)  $\frac{d}{dx}x^3 + 3x^2 - 4 = 3x^2 + 6x$   
 (b)  $\frac{d^2}{dx^2}x^3 + 3x^2 - 4 = 6x + 6$   
 (c)  $\frac{d}{dx}3e^x = 3e^x$   
 (d)  $\frac{d}{dx}3e^x + e^3 + x^e = 3e^x + 0 + x^e(\ln e) = 3e^x + x^e$   
 (e)  $\frac{d}{dx}\pi^3 = 0$

3. Let

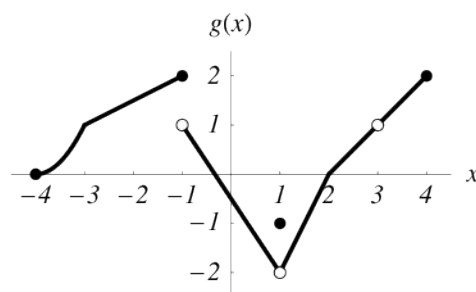
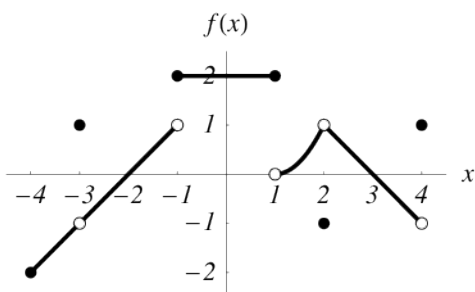
$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 0 \\ ax + b & \text{if } x \geq 0 \end{cases}$$

Find all the values  $a$  and  $b$  such that

- (a)  $f(x)$  is continuous.  
 (b)  $f(x)$  is differentiable.

**Answer:** (a)  $b = 1$  and  $a$  can be any real number.  
 (b)  $a = 0$  and  $b = 1$

4. Given the graphs of  $f$  and  $g$  below, evaluate each of the following limits. If the limit does not exist, say why.



- (a)  $\lim_{x \rightarrow 2} f(x)$

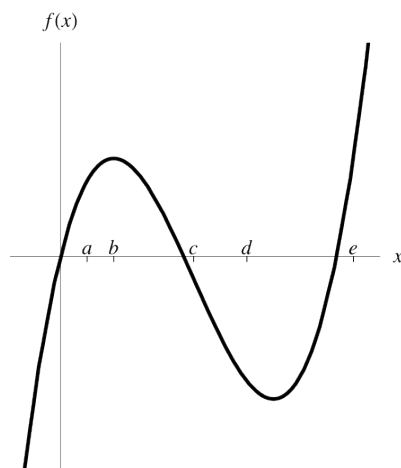
- (b)  $\lim_{x \rightarrow -1^+} g(x)$   
 (c)  $\lim_{x \rightarrow -1^-} g(x)$   
 (d)  $\lim_{x \rightarrow -1} [f(x) + g(x)]$   
 (e)  $\lim_{x \rightarrow 3} \frac{f(x)}{x-3}$   
 (f)  $\lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$   
 (g)  $\lim_{x \rightarrow -2} \frac{g(x) - g(-2)}{x+2}$

**Answer:**

(a) 1    (b) 2    (c) 1    (d) DNE    (e) -1 or at least know that  $\frac{0}{0}$  means you need to do more more work to figure it out. (f) -1    (g)  $\frac{1}{3}$

5. The graph of  $f$  is shown below. Match the derivatives in the following table with the points  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$  on the graph below.

$x$	0	1.75	6.75	-1.25	-2.25
$f'(x)$					



**Answer:**

$x$	$b$	$a$	$e$	$d$	$c$
$f'(x)$	0	1.75	6.75	-1.25	-2.25

6. Use the table of values for  $f$ ,  $g$ ,  $f'$ , and  $g'$  to answer the following questions.

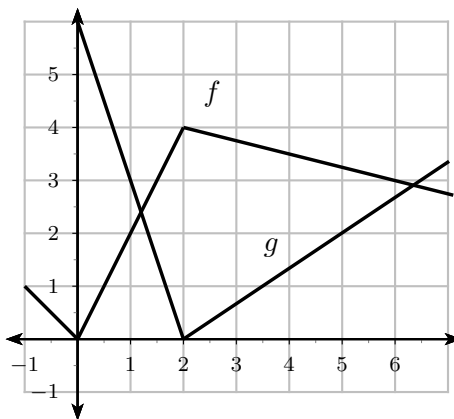
$x$	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

- (a) If  $h(x) = f(g(x))$ , find  $h'(1)$ .

(b) If  $H(x) = g(f(x))$ , find  $H'(1)$ .

**Answer:** (a)  $h'(1) = 30$     (b)  $H'(1) = 8$

7. If  $f$  and  $g$  are the functions whose graphs are shown, let  $u(x) = f(g(x))$ ,  $v(x) = g(f(x))$ , and  $w(x) = g(g(x))$ .



Find each of the following derivatives.

(a)  $u'(1)$

(b)  $v'(1)$

(c)  $w'(1)$

**Answer:** (a)  $u'(1) = \frac{3}{4}$     (b)  $v'(1)$  DNE    (c)  $w'(1) = -2$