Math 510

Exam Review 2

1. Determine the following algebraically (without a calculator)

   (a) $\lim_{x\to 3} \dfrac{x^2-9}{x^2-4x+3}$

   (b) $\lim_{x\to 3} \dfrac{\frac{1}{x}-\frac{1}{3}}{x-3}$

   (c) $\lim_{x\to \infty} \dfrac{2x^2-1}{x^2+2x+1}$

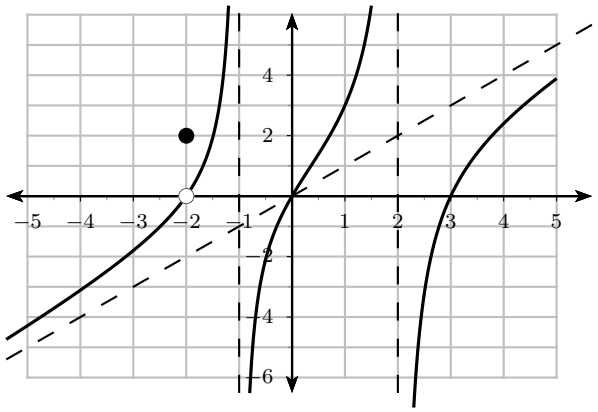
   (d) $\lim_{x\to \infty} \sin x$

   (e) $\lim_{x\to \infty} \dfrac{x^3+3x}{x^4-1}$

   (f) $\lim_{x\to \infty} \dfrac{x^2}{x+1}$

Answer:
1. (a) 3 (b) $\frac{-1}{9}$ (c) 2 (d) DNE (e) 0 (f) ∞

2. Use the graph of f below to evaluate the following limits.



   (a) $\lim_{x\to -2} f(x)$

   (b) $\lim_{x\to -1} f(x)$

   (c) $\lim_{x\to 0} f(x)$

   (d) $\lim_{x\to 2^-} f(x)$

   (e) $\lim_{x\to 2^+} f(x)$

   (f) $\lim_{x\to 2} f(x)$

   (g) $\lim_{x\to -\infty} f(x)$

Answer: 2. (a) 0 (b) DNE (c) 0 (d) ∞ (e) $-\infty$ (f) DNE (g) $-\infty$

3. Sketch a graph of some function f that satisfies all the following properties.

The domain of f is $[-5, 5]$.

$f(-5) = 0$, $f(-3) = 2$, $f(-1) = 0$, $f(0) = 0$, $f(1) = 0$, $f(3) = -2$, $f(5) = -4$

$\lim_{x \to -5^+} f(x) = +\infty$, $\lim_{x \to -3} f(x) = 0$, $\lim_{x \to -1^-} f(x) = +\infty$, $\lim_{x \to -1^+} f(x) = -\infty$

$\lim_{x \to 0} f(x) = 0$, $\lim_{x \to 1} f(x) = +\infty$, $\lim_{x \to 3} f(x) = 0$, $\lim_{x \to 5^-} f(x) = -\infty$

Answer: Many possible answers

4. Let

$$f(x) = \begin{cases} x^2 - 4, & \text{if } x \leq 2 \\ 2 - x, & \text{if } 2 < x \leq 4 \\ x - 2, & \text{if } 4 < x \end{cases}$$

   (a) $\lim_{x \to 2^-} f(x)$

   (b) $\lim_{x \to 2^+} f(x)$

   (c) $\lim_{x \to 2} f(x)$

   (d) $\lim_{x \to 4^-} f(x)$

   (e) $\lim_{x \to 4^+} f(x)$

   (f) $\lim_{x \to 4} f(x)$

Answer: 4. (a) 0 (b) 0 (c) 0 (d) -2 (e) 2 (f) DNE

5. Determine the value of a and b such that f will be continuous for all x.

$$f(x) = \begin{cases} 2x + 1, & \text{if } x \leq 3 \\ ax + b, & \text{if } 3 < x < 5 \\ x - 2, & \text{if } 5 \leq x \end{cases}$$

Answer: $a = -2$ and $b = 13$

6. Find all the derivatives of $f(x) = x^4 + 3x^3 - x^2 + 4$

Answer: $f'(x) = 4x^3 + 9x^2 - 2x$
$f''(x) = 12x^2 + 18x - 2$
$f'''(x) = 24x + 18$
$f^{(4)}(x) = 24$
$f^{(5)}(x) = 0$

7. Determine the derivative of the following.

   (a) $y = \cos x \cot x$

   (b) $y = \dfrac{\sin x}{1 - \cos x}$

(c) $y = \tan(3t^2 + 2t)$

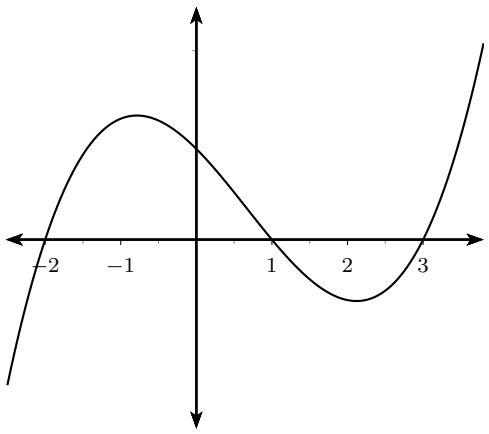
(d) $y = \sin(\cos x)$

Answer:

6. (a) $y' = -\sin x \cot x - \cos x \csc^2 x$ (b) $y' = \dfrac{1}{\cos x - 1}$

(c) $y' = \sec^2(3t^2 + 2t) \cdot (6t + 2)$ (d) $y' = \cos(\cos x)[-\sin x]$

8. Use the graph of f', the **derivative of** f, below to answer the following questions.



(a) Determine the intervals on which f is increasing.

(b) Determine the $x-$coordinate(s) of all local minima of f.

(c) Determine whether f is concave up, concave down, or neither at $x = 0$. Justify your answer.

(d) If $f(0) = 0$, sketch a possible graph of f.

Answer: (a) $-2 < x < 1$ and $3 < x$ (b) $x = -2$ and 3 (c) Since $f''(x) < 0$ at $x = 0$, the graph of f is concave down. (d) many possible answers.