Math 510

Exam Review 1

1. Determine the second derivative of the following.
   (a) $f(x) = \sqrt{x^2+9}$

   Answer: $f'(x) = \dfrac{9}{(x^2+9)^{3/2}}$

   (b) $f(t) = \dfrac{t}{(1-t)^2}$

   Answer: $f'(t) = \dfrac{2(t+2)}{(1-t)^4}$

   (c) $f(\theta) = \cot x$

   Answer: $f'(\theta) = 2\csc^2\theta\cot\theta$

2. For $y = \sqrt[3]{(x-2)^2}$ determine
   (a) the equation of the tangent line at $(3,1)$

   Answer: $y - 1 = \frac{2}{3}(x-3)$

   (b) the equation of the normal line at $(3,1)$.

   Answer: $y - 1 = \frac{-3}{2}(x-3)$

3. Find the points on the graph of

$$f(x) = \frac{x^3}{3} + x^2 - x - 1$$

   at which the slope is
   (a) -1

   Answer: $(0,-1)$ and $(-2,\frac{7}{3})$

   (b) 2

   Answer: $(-3,2)$ and $(1,-\frac{2}{3})$

   (c) 0

   Answer: $x = -1 \pm \sqrt{2}$

4. Determine the following limits.
   (a) $\displaystyle\lim_{x\to 0} \frac{\frac{1}{x+1} - 1}{x}$

Answer: -1

(b) $\lim_{x \to -2} \dfrac{t+2}{t^2-4}$

Answer: $\frac{-1}{4}$

5. Determine the value of a so that the following function is continuous on the entire real line.

$$f(x) = \begin{cases} x+3, & \text{if } x \leq 2 \\ ax+6, & \text{if } x > 2 \end{cases}$$

Answer: $a = -\frac{1}{2}$

6. Use the limit definition of the derivative to find y' if $y = \dfrac{1}{x}$

Answer:

$$\begin{aligned} \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} &= \lim_{h \to 0} \frac{\frac{x-(x+h)}{x(x+h)}}{h} \\ &= \lim_{h \to 0} \frac{\frac{-h}{x(x+h)}}{h} \\ &= \lim_{h \to 0} \frac{-1}{x(x+h)} \\ &= -\frac{1}{x^2} \end{aligned}$$