Math 510

Product and Quotient Rules

§3.2

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(6) 1. Find the points on the graph of $y = x^{3/2} - x^{1/2}$ at which the tangent line is parallel to the line $y - x = 3$.

Solution: $y'(x) = \frac{3}{2}\sqrt{x} - \frac{1}{2}\cdot\frac{1}{\sqrt{x}}$. We want to find the points on y that have a slope of 1. So, we want to know when $y'(x) = 1$.

$$\begin{aligned}
1 &= \frac{3}{2}\sqrt{x} - \frac{1}{2}\cdot\frac{1}{\sqrt{x}} \\
2 &= 3\sqrt{x} - \frac{1}{\sqrt{x}} \\
2\sqrt{x} &= 3x - 1 \\
4x &= 9x^2 - 6x + 1 \\
0 &= 9x^2 - 10x + 1 \\
0 &= (9x-1)(x-1)
\end{aligned}$$

So, $x = \frac{1}{9}$ and $x = 1$. But a quick check shows that $y'\left(\frac{1}{9}\right) = -1$. So, the only point is $x = 1$.

(6) 2. Find the equation of the tangent line to the curve $y = \frac{e^x}{1+x^2}$ at the point $\left(1, \frac{e}{2}\right)$.

Solution:

$$\begin{aligned}
y'(x) &= \frac{(1+x^2)e^x - e^x(2x)}{(1-x^2)^2} \\
&= \frac{e^x x^2 - 2e^x x + e^x}{x^4 + 2x^2 + 1}
\end{aligned}$$

and $y'(1) = \frac{(e)1^2 - 2e(1) + e}{1^4 + 2^2 + 1} = \frac{0}{4} = 0$. So, the equation of the tangent line is $y - \frac{e}{2} = 0(x-1)$ or $y = \frac{e}{2}$

(6) 3. If f and g are functions such that $f(2) = 3$, $f'(2) = -1$, $g(2) = -5$, and $g'(2) = 2$, find $\left(\frac{f}{g}\right)'(2)$.

Solution:

$$\begin{aligned}
\left(\frac{f}{g}\right)'(2) &= \frac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2} \\
&= \frac{-5(-1) - 3(2)}{(-5)^2} \\
&= \frac{-1}{25}
\end{aligned}$$

(6) 4. Find the second derivative of $y = \dfrac{3x-1}{\sqrt{x}}$.

Solution:

$$\begin{aligned} y'(x) &= \frac{\sqrt{x}(3) - (3x-1)\frac{1}{2}x^{-\frac{1}{2}}}{x} \\ &= \frac{3x+1}{2\sqrt{x^3}} \\ y''(x) &= \frac{-3\sqrt{x^3} - 3\sqrt{x}}{4x^3} \end{aligned}$$