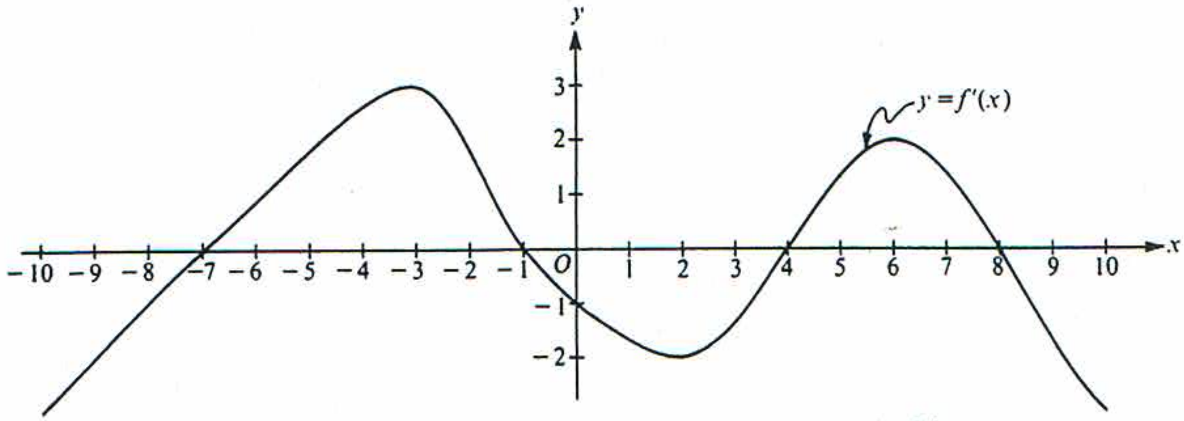
Math 510

More What does f' say about f?

§2.9

1.

1989 – AB 5



Note: This is the graph of the derivative of f, not the graph of f.

The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-10 \leqq x \leqq 10$.

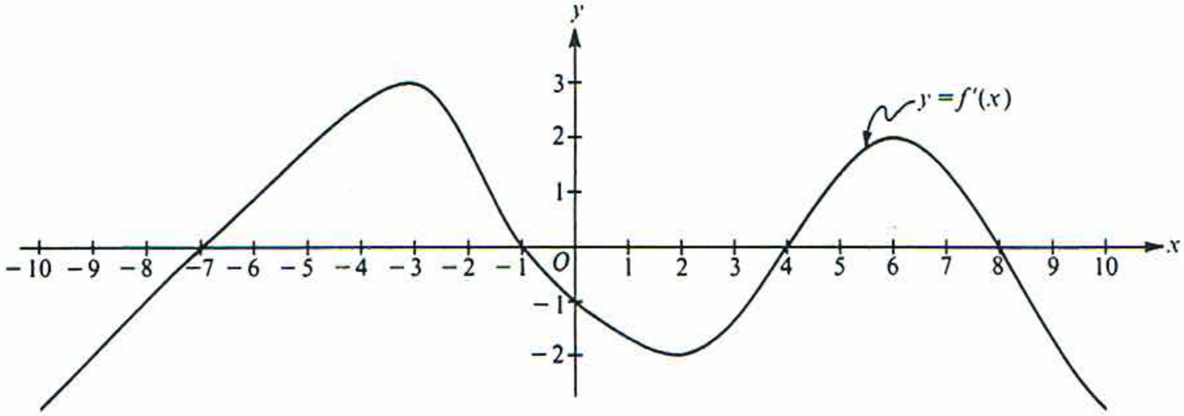
(a) For what values of x does the graph of f have a horizontal tangent?

(b) For what values of x in the interval $(-10, 10)$ does f have a relative maximum? Justify your answer.

(c) For what values of x is the graph of f concave downward?

Solution:

1989 – AB 5



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a.) f has a horizontal tangent at points where $f'(x) = 0$. This occurs at $x = -7, -1, 4, 8$

b.)

	−10 to −7	−7 to −1	−1 to 4	4 to 8	8 to 10
$f'(x)$:	−	+	−	+	−
f:	decr.	incr.	decr.	incr.	decr.

f has a relative max. at $x = -1$ and at $x = 8$

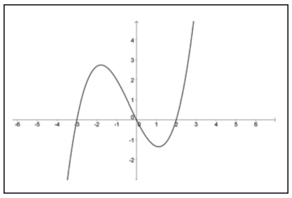
$$\left.\begin{array}{l} f \text{ continuous at } x = a \\ f \text{ increasing when } x < a \\ f \text{ decreasing when } x > a \end{array}\right\} \Rightarrow f(a) \text{ is a relative max.}$$

c.)

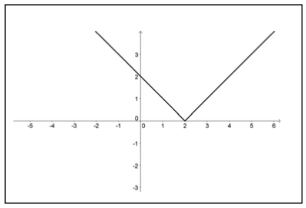
	−10 to −3	−3 to 2	2 to 6	6 to 10
$f''(x)$:	+	−	+	−

f is concave down when $-3 < x < 2$ or $6 < x < 10$

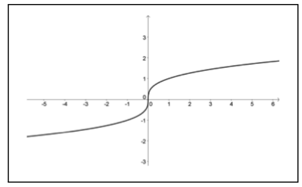
2. The graphs of four functions (a)-(d) are shown. Match each one with its derivative, chosen from the six graphs (e)-(j) pictured below.



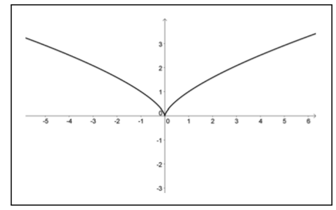
(a)



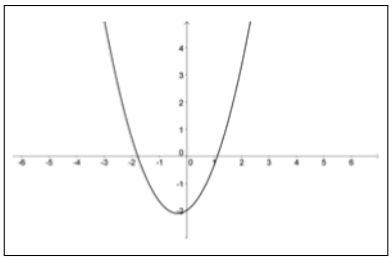
(b)



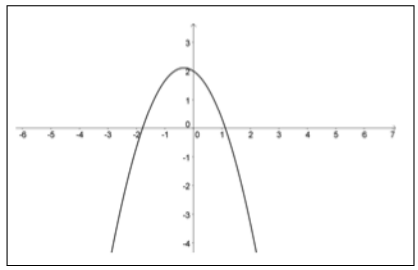
(c)



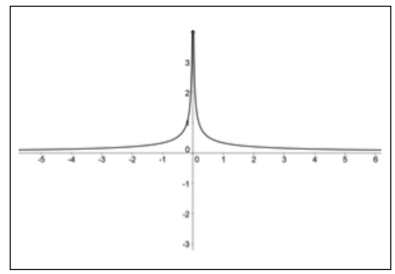
(d)



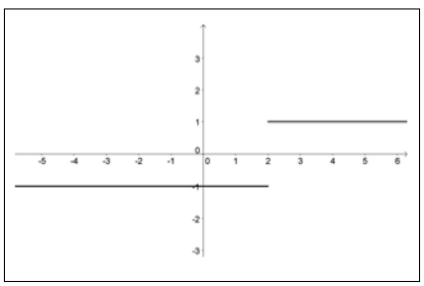
(e)



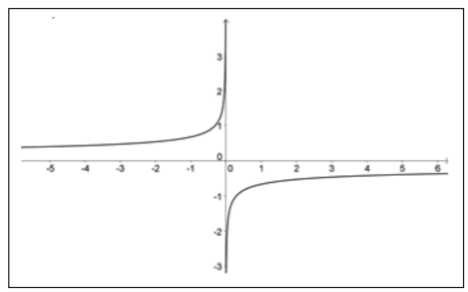
(f)



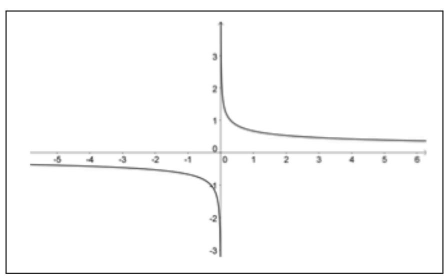
(g)



(h)



(i)



(j)

Solution: (a)-(e) (b)-(h) (c)-(g) (d)-(j)

3. The graph of f' is shown below. Use it to answer the following questions.

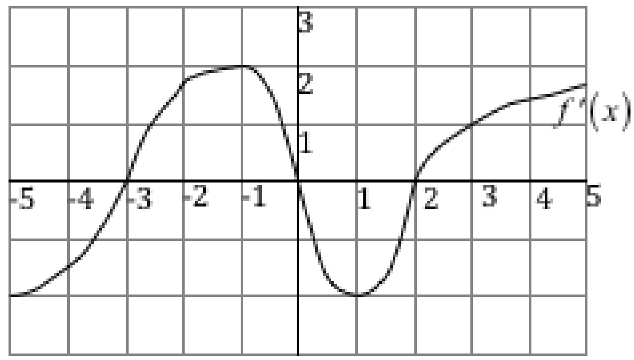


Figure 1: default

(a) On what interval(s) is f increasing?

Solution: $-3 < x < 0$ and $2 < x < 5$

(b) On what interval(s) is f concave down?

Solution: $-1 < x < 1$

(c) Identify, if any, the $x-$coordinate of all local maxima and minima of f.

Solution: Local max at $x = 0$, Local min at $x = -3$ and 2

(d) Identify, if any, the $x-$coordinate of all points of inflection of f.

Solution: $x = -1$ and 1

(e) If $f(1) = 0$, is $f(2)$ positive or negative? Justify.

Solution: $f(2) < 0$ because the values of f' are negative for all x between 1 and 2 so the graph of f is decreasing for $1 < x < 2$. Since $f(1) = 0$, $f(2) < 0$.