Math 510

Derivative of Trig Functions

§3.4

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$
$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\cot x = -\csc^2 x$$
$$\frac{d}{dx}\sec x = (\sec x)(\tan x) \qquad \frac{d}{dx}\csc x = -(\csc x)(\cot x)$$

1. Determine y' for each of the following.

   (a) $y = \sin x - \cos x$

   Solution: $y' = \cos x + \sin x$

   (b) $y = \dfrac{\tan x}{x+1}$

   Solution: $y' = \dfrac{(x+1)(\sec^2 x) - (\tan x)(1)}{(x+1)^2} = \dfrac{x\sec^2 x + \sec^2 x - \tan x}{(x+1)^2}$

   (c) $y = \sin\frac{\pi}{4}$

   Solution: $y' = 0$

   (d) $y = x^3 \sin x$

   Solution: $y' = x^3(\cos x) + (\sin x)(3x^2) = x^3\cos x + 3x^2\sin x$

   (e) $y = x^2 + 2x\cos x$

   Solution: $y' = 2x + (2x)(-\sin x) + (\cos x)(2) = 2x - 2x\sin x + 2\cos x = 2(x - x\sin x + \cos x)$

   (f) $y = \dfrac{x}{\sec x + 1}$

   Solution: $y' = \dfrac{(\sec x + 1)(1) - x(\sec x)(\tan x)}{(\sec x + 1)^2} = \dfrac{\sec x + 1 - x\sec x\tan x}{(\sec x + 1)^2}$

   (g) $y = \dfrac{x}{\cot x}$

   Solution: $y' = \dfrac{(\cot x)(1) - (x)(-\csc^2 x)}{(\cot x)^2}$

2. Find where the graph of $f(x) = \sqrt{3}\sin x + 3\cos x$ has a horizontal tangent.

Solution: Solve $f'(x) = \sqrt{3}\cos x - 3\sin x = 0$.

$$\begin{aligned} \sqrt{3}\cos x - 3\sin x &= 0 \\ \sqrt{3}\cos x &= 3\sin x \\ \frac{\sin x}{\cos x} &= \frac{\sqrt{3}}{3} \\ \tan x &= \frac{\sqrt{3}}{3} \end{aligned}$$

So, the horizontal tangents are when $x = \frac{\pi}{6} + k\pi$, where $k \in \mathbb{Z}$

3. Determine the 43 derivative of $y = \sin x$

Solution: The 43 derivative of $\sin x$ is $-\cos x$.

4. Find the equation of the tangent line to $y = \sec x$ at
   (a) $x = \frac{\pi}{4}$

   Solution: $y - \sqrt{2} = \sqrt{2}(x - \frac{\pi}{4})$

   (b) $x = -\frac{\pi}{4}$

   Solution: $y - \sqrt{2} = -\sqrt{2}(x + \frac{\pi}{4})$

   (c) Find the point of intersection of the lines from (a) and (b).

   Solution: $(0, 0.3035)$