Math 510

The Product and Quotient Rules

§3.2

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x) \cdot \frac{d}{dx}g(x) + g(x) \cdot \frac{d}{dx}f(x)$$

"The derivative of a product is the first times the derivative of the second plus the second times the derivative of the first"

1. Determine the following derivatives and you do not need to simplify.

   (a) $\dfrac{d}{dx}x^3e^x$

   > **Solution:** $\dfrac{d}{dx}x^3e^x = x^3e^x + e^x(3x^2)$

   (b) $d\frac{d}{dx}\sqrt{x}e^x$

   > **Solution:** $\dfrac{d}{dx}\sqrt{x}e^x = \sqrt{x}e^x + e^x(\frac{1}{2}x^{3/2}$

The Quotient Rule If f and g are differentiable, then

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x) \cdot \frac{d}{dx}f(x) - f(x) \cdot \frac{d}{dx}g(x)}{[g(x)]^2}$$

"the derivative of a quotient is the bottom times the derivative of the top minus the top times the derivative of the bottom all over the bottom squared"

2. Determine the following derivatives and you do not need to simplify.

   (a) $\dfrac{d}{dx}\dfrac{4x-5}{3x+2}$

   > **Solution:** $\dfrac{d}{dx}\dfrac{4x-5}{3x+2} = \dfrac{(3x+2)(4)-(4x-5)(3)}{(3x+2)^2}$

   (b) $\dfrac{d}{dx}\dfrac{7}{x^2+5}$

   > **Solution:** $\dfrac{d}{dx}\dfrac{7}{x^2+5} = \dfrac{(x^2+5)(0)-7(2x)}{(x^2+5)^2} = \frac{-14x}{(x^2+5)^2}$

   (c) $\dfrac{d}{dx}\dfrac{3x^5+2x}{5}$

   > **Solution:** $\dfrac{d}{dx}\dfrac{3x^5+2x}{5} = \dfrac{5(15x^4+2)-(3x^5+2x)(0)}{5^2} = \frac{5(15x^4+2)}{25}$

3. For, $f(x) = 6x^4 + 24x^3 - 540x^2 + 7$, solve $f''(x) = 0$.

Solution: $f'(x) = 24x^3 + 72x^2 - 1080x$ and $f''(x) = 72x^2 + 148x - 1080$.
The solution of $72x^2 + 148x - 1080 = 0$ are found using your calculator to be $x = -5.035$ and $x = 2.979$.

4. For $f(x) = \dfrac{3x-1}{x^2}$ find $f'(x)$ three ways; (a) using the product rule, (b) using the quotient rule, and (c) simplifying algebraically and using the general power rule.

Solution: (a) Using the product we first need to write this as a product. So,$f(x) = \frac{3x-1}{x^2} = (3x-1)(x^{-2}$. Then

$$\begin{aligned} f'(x) &= (3x-1)(-2x^{-3}) + x^{-2}(3) \\ &= -x^{-2} + 2x^{-3} + 3x^{-2} \\ &= \frac{-6}{x^2} + \frac{2}{x^3} + \frac{3}{x^2} \\ &= \frac{2}{x^3} - \frac{3}{x^2} \end{aligned}$$

(b) Using the quotient rule we have

$$\begin{aligned} f'(x) &= \frac{x^2(3) - (3x-1)(2x)}{(x^2)^2} \\ &= \frac{3x^2 - 6x^+2x}{x^4} \\ &= \frac{-3x^2 + 2x}{x^4} \\ &= \frac{x(-3x+2}{x^4} \\ &= \frac{2-3x}{x^3} = \frac{2}{x^3} - \frac{3}{x^2} \end{aligned}$$

(c) Using the general power rule we first need to simplify. So,

$$f(x) = \frac{3x-1}{x^2} = (3x-1)(x^{-2} = (3x-1)(x^{-2}) = 3x^{-1} - x^{-2}$$

$$f'(x) = -3x^{-2} + 2x^{-3} = frac{-}3x^2 + \frac{2}{x^{-3}}$$

5. Find the coordinates of all points on the graph of $f(x) = x^3 + 2x^2 - 4x + 5$ at which the tangent line is

(a) horizontal

Solution: The tangent line to f will be horizontal when $f'(x) = 0$. So, $f'(x) = 3x^2 + 4x - 4 = 0$ when $x = \frac{2}{3}$ and $x = -2$.

(b) parallel to the line $2y + 8x = 5$.

Solution:

(c) The slope of $2y+8x=5$ is $m=-4$. So we need to solve $f'(x)=-4$. Using your calculator we get $x=0$ and $x=-\frac{4}{3}$.