Math 510

Derivatives of Polynomials & Exponential Functions

§3.1

This is how to calculate the derivative for many types of functions using formulas rather than the limit definition. We'll start off with the derivatives of a constant function, polynomials, power functions, sums and differences, and finally exponential functions.
The derivative of a **constant** function $f(x) = c$ is

$$\frac{d}{dx}c = 0$$

The **General Power Rule** states:

$$\frac{d}{dx}x^n = nx^{n-1}$$

The derivative of an arithmetic combination of functions can be found by putting together the derivatives of the components using these rules:

$$\frac{d}{dx}\left[cf(x)\right] = c\frac{d}{dx}f(x) \quad \text{for any constant } c$$

"the derivative of a constant times a function is the constant time the derivative of the function"

$$\frac{d}{dx}\left[f(x) + g(x)\right] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

"the derivative of a sum of a function is the sum of the derivatives"

$$\frac{d}{dx}\left[f(x) - g(x)\right] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

"the derivative of a difference of a function is the difference of the derivatives"

1. Determine the derivative of the following.

   (a) $\dfrac{d}{dx} - 55$

   Solution: $\dfrac{d}{dx} - 55 = 0$

   (b) $\dfrac{d}{dx}x^6$

   Solution: $\dfrac{d}{dx}x^6 = 6x^5$

   (c) $\dfrac{d}{dx}5x^4$

   Solution: $\dfrac{d}{dx}5x^4 = 20x^3$

   (d) $\dfrac{d}{dx}(x^4 + 4x^3)$

Solution: $\frac{d}{dx}(x^4 + 4x^3) = 4x^3 + 12x^2$

(e) $\frac{d}{dx}(7x^3 + 6x^2 + 10x + 12)$

Solution: $\frac{d}{dx}(7x^3 + 6x^2 + 10x + 12) = 21x^2 + 12x + 10$

(f) $\frac{d}{dx}(x^{-7} - x^{-6})$

Solution: $\frac{d}{dx}(x^{-7} - x^{-6}) = -7x^-6 + 6x^-5$

(g) $\frac{d}{dx}x^{\pi}$

Solution: $\frac{d}{dx}x^{\pi} = \pi x^{\pi-1}$

2. Suppose the position of a particle at any time t is given by the following function

$$s(t) = t^2 + \frac{1}{t}.$$

Find the velocity of the particle at time $t = 2$seconds.

Solution: $v(t) = s'(t) = 2t - t^{-2}$ and $v(2) = 4 - \frac{1}{4} = \frac{15}{4}$.

3. For $f(x) = mx + b$, find $f'(x)$.

Solution: $f'(x) = m$ the slope of the line

4. Find the equation of the line tangent to the curve $f(x) = x^3 - 6x^2$ at its point of inflection.

Solution: $f'(x) = 3x^2 - 12x$ and$f''(x) = 6x - 12$. So the possible inflection point is when $f''(x) = 0$ as long as it chages sign. Which is does, so $x = 2$ is the inflection point.
Then, $f'(2) = -12$ and $f(2) = -16$ so the equation of the tangent line is $y + 16 = -12(x - 2)$.

Exponential functions have the form

$$f(x) = a^x.$$

Because the base is a constant and the exponent is a variable, its derivative cannot be found using the General Power Rule. Instead, for $f(x) = a^x$ we have the following (For the proof of this see page 188 of your text).

$$f'(x) = a^x \cdot f'(0)$$

"the derivative of an exponential function is proportional to the function itself"

If we define

$$\left.\frac{d}{dx}e^x\right|_{x=0} = \lim_{h\to 0}\frac{e^h - 1}{h} = 1$$

then when $f(x) = e^x$ we have $f'(0) = 1$. Therefore

$$\frac{d}{dx}e^x = e^x$$

5. Determine the derivatives of the following.

   (a) $\frac{d}{dx}(3e^x + x^3 + 3x)$

   Solution: $\frac{d}{dx}(3e^x + x^3 + 3x) = 3e^x + 3x^2 + 3$

   (b) $\frac{d}{dx}(3e^x + \pi)$

   Solution: $\frac{d}{dx}(3e^x + \pi) = 3e^x$

6. Find the equation of the line tangent to the curve $f(x) = e^x$ at $x = 0$ and sketch both on the same axis.

Solution: $f'(0) = 1$ and $f(0) = 1$. So, the equation of the tangent line is $y - 1 = 1(x - 0)$.