Math 510

Derivatives as a Function 2

§2.8

Like continuity, differentiability can be considered from the left or from the right.
If a function is **left differentiable at** $x = a$ then the following limit exists.

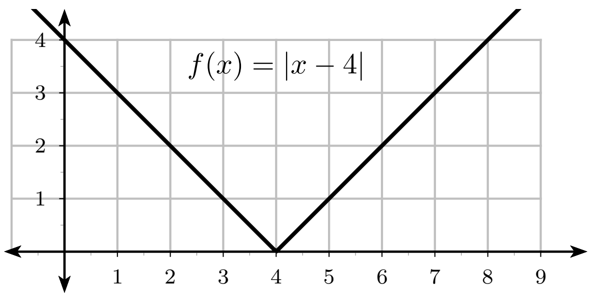
$$f'_-(a) = \lim_{x \to a^-} \frac{f(x) - f(a)}{x - a} \quad \text{or} \quad f'_-(a) = \lim_{h \to 0^-} \frac{f(a+h) - f(a)}{h}$$

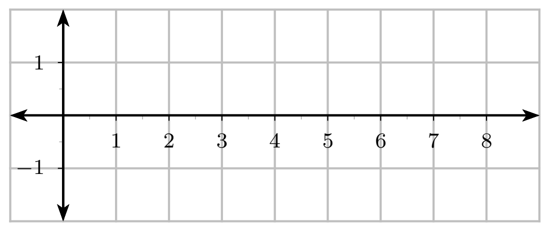
If a function is **right differentiable at** $x = a$ then the following limit exists.

$$f'_+(a) = \lim_{x \to a^+} \frac{f(x) - f(a)}{x - a} \quad \text{or} \quad f'_+(a) = \lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$$

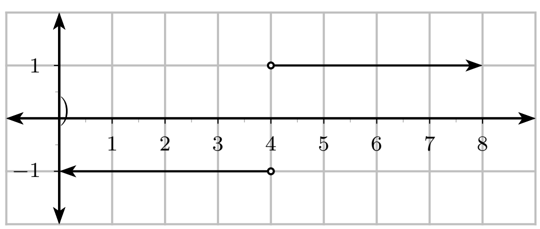
These are saying "What is the slope of the curve immediately to the left and right of a."

1. Use the graph of $f(x) = |x - 4|$ on the left to answer the following questions.





   (a) Sketch the graph of f' on the axes to the right.



Solution:

   (b) From the graph of f', determine the value of $f'_-(4)$.

Solution: $f'_-(4) = 1$

   (c) From the graph of f', determine the value of $f'_+(4)$

Solution: $f'_+(4) = -1$

   (d) From the graph of f', determine the value of $f'(4)$

Solution: undefined because the derivative from the left is not the same as the derivative from the right.

2. Consider the piecewise defined function

$$f(x) = \begin{cases} 3x + 2, & \text{if } x < 1 \\ 6 - x, & \text{if } x \geq 1 \end{cases}$$

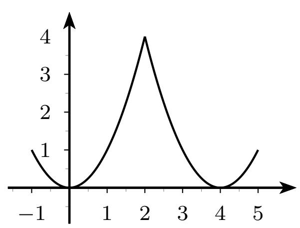
(a) Determine $f'_{-}(1)$ and $f'_{+}(1)$

Solution: $f'_{-}(1) = 3$ and $f'_{+}(1) = -1$

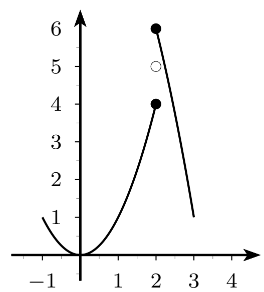
(b) Write down a formula for $f'(x)$ as a piecewise defined function.

Solution: $f(x) = \begin{cases} 3, & \text{if } x < 1 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x \geq 1 \end{cases}$

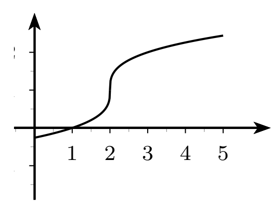
There are three common ways for a function to fail to be differentiable at a point.



This graph has a sharp corner.



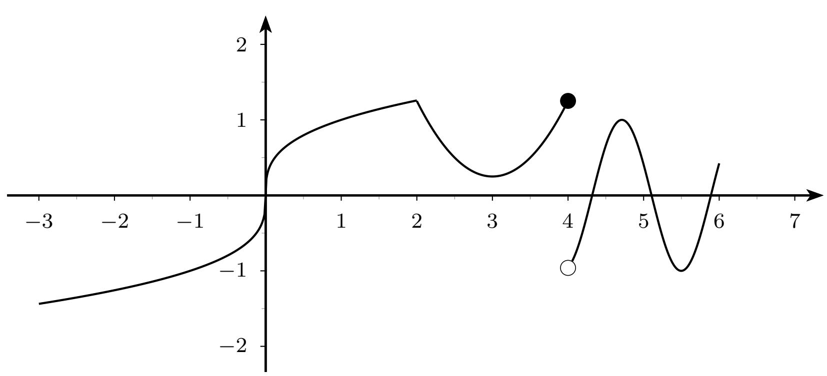
This graph is not continuous.



This graph has a vertical tangent.

If f is differentiable at a point c, then f is also continuous at c. This means that if you draw a tangent line to a graph, then the graph must be unbroken at that point. The converse is false: continuity does not imply differentiability.

3. Use the graph below to answer the following questions either TRUE or FALSE.



(a) $f(x)$ is continuous 0.

Solution: True

(b) $f(x)$ is differentiable at 0.

Solution: False

(c) $f(x)$ is continuous 2.

Solution: True

(d) $f(x)$ is differentiable at 2.

Solution: False

(e) $f(x)$ is continuous 3.

Solution: True

(f) $f(x)$ is differentiable at 3.

Solution: True

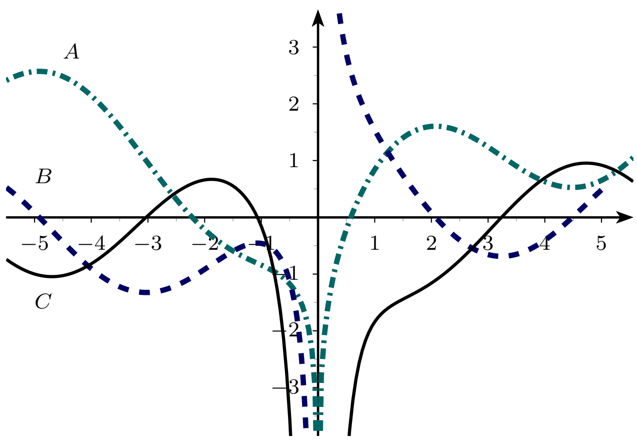
(g) $f(x)$ is continuous 4.

Solution: False

(h) $f(x)$ is differentiable at 4.

Solution: False

4. Identify the original function, first derivative, and second derivative.



original function:
first derivative:
second derivative:

Solution: original function: A first derivative: B second derivative:C