Math 510

Derivative as a Function 1

§2.8

By varying x, $f'(x)$ defines a function called **the derivative of f with respect to x:**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}. \tag{1}$$

Using your `TI-84`, you can find both $f'(a)$ and $f'(x)$.

$$f'(a) \;\Leftrightarrow\; \texttt{nDeriv(function, x, a)} \qquad\qquad f'(x) \;\Leftrightarrow\; \texttt{nDeriv(function, x, x)}$$

or

$$f'(a) \;\Leftrightarrow \frac{d}{d(x)}\,(function)\,|_{x=a} \qquad\qquad f'(x) \;\Leftrightarrow \frac{d}{d(x)}\,(function)\,|_{x=x}$$

Push `MATH` and look under the `MATH Menu` for `8:nDeriv(` .

The function f is **differentiable on an interval** if it is differentiable at every number in the interval (including the endpoints if necessary). All of these notations are used to refer to the derivative of f at x:

$$f'(x), \qquad y', \qquad \frac{df}{dx}, \qquad \frac{dy}{dx}, \qquad \frac{d}{dx}f(x), \qquad Df(x), \qquad D_x f(x)$$

1. Let $f(x) = \sqrt{5-x}$.

   (a) Complete the following table.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f'(x)$											

Solution:

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f'(x)$	-.1581	-.1667	-.1768	-.189	-.2041	-.2236	-.25	-.2887	-.3536	-.5	error?(Why)

   (b) Determine the equation of the tangent line to f at $x = 2$.

Solution: $y - \sqrt{3} = -.2887(x-2)$

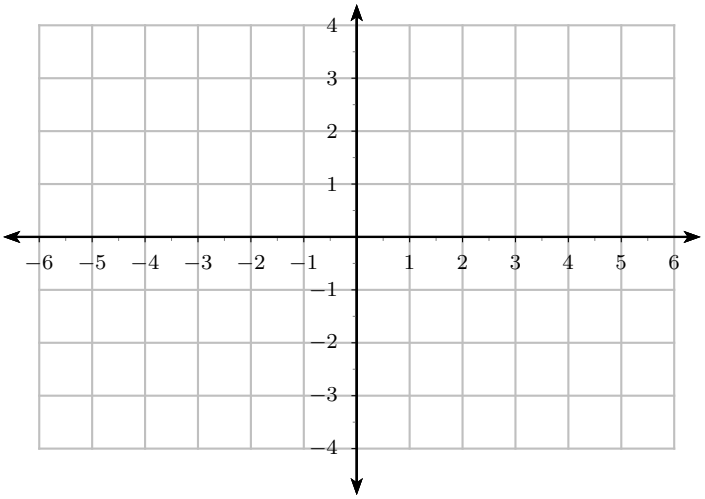
   (c) Complete the following table.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f''(x)$											

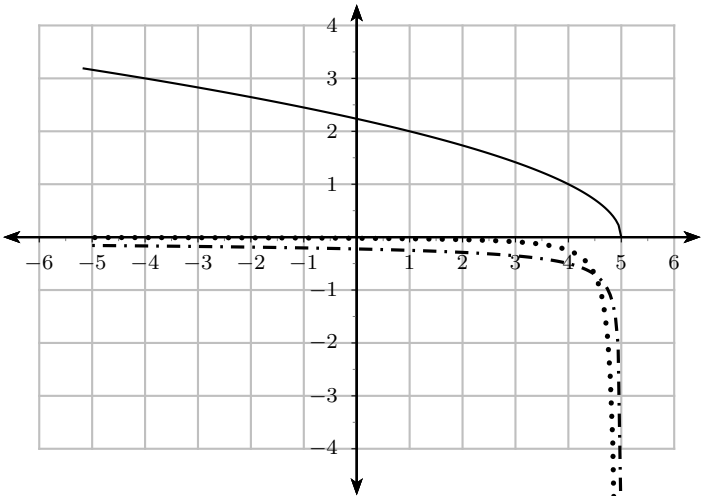
Solution:

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f''(x)$	-.0079	-.0093	-.011	-.0135	-.017	-.0224	-.0313	-.0481	-.0884	-.25	error?(Why?)

   (d) Sketch the graph of f, f', and f'' on the axes.

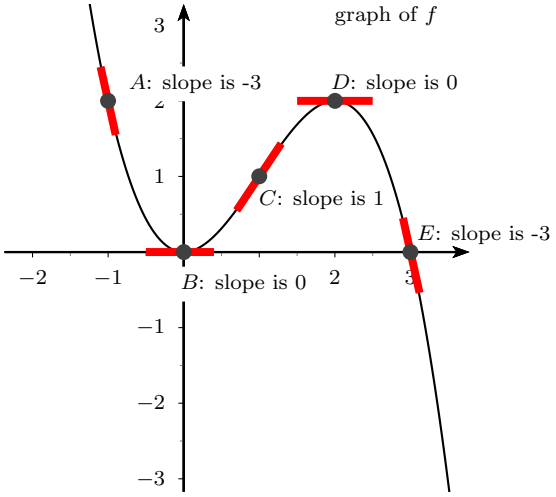


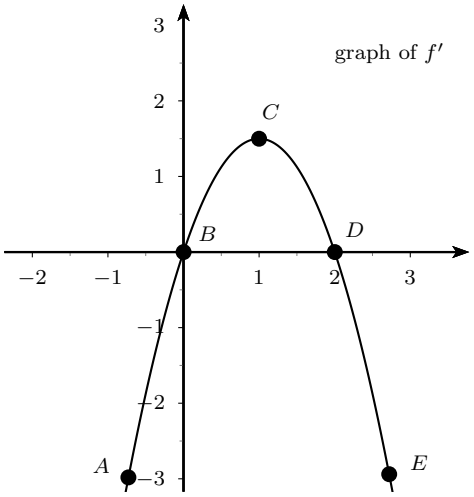
Solution:



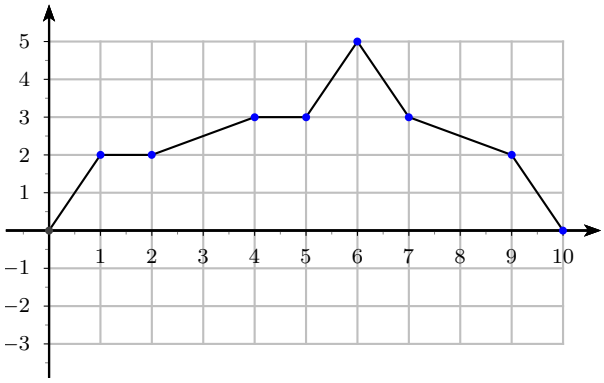
The graph of f' may be determined from the graph of f by remembering that $f'(x)$ is the slope of the tangent line at $(x, f(x))$:

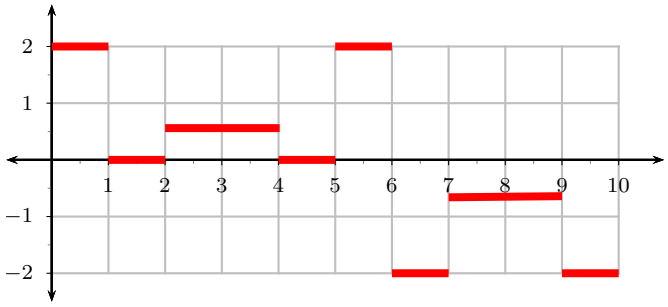
- If c is the slope of the tangent line at $(x, f(x))$, the (x, c) is on the graph of $f'(x)$.
- Conversely, if (x, c) is on the graph of f', then $f'(x) = c$ and c is the slope of the tangent line to f at $(x, f(x))$.





2. The graph of f is sketched below. Sketch f' on the axis to the right.





3. The graph of f is on the left. Sketch the graph of f' on the axis below it. Use a sign chart from the tutorial to help.