

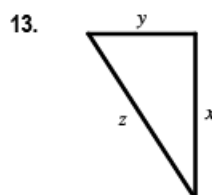
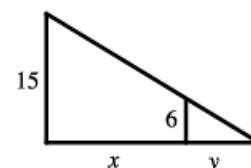
12. (a) Given: a man 6 ft tall walks away from a street light mounted on a 15-ft-tall pole at a rate of 5 ft/s. If we let t be time (in s) and x be the distance from the pole to the man (in ft), then we are given that $dx/dt = 5$ ft/s.

(b) Unknown: the rate at which the tip of his shadow is moving when he is 40 ft from the pole. If we let y be the distance from the man to the tip of his shadow (in ft), then we

want to find $\frac{d}{dt}(x + y)$ when $x = 40$ ft.

(d) By similar triangles, $\frac{15}{6} = \frac{x + y}{y} \Rightarrow 15y = 6x + 6y \Rightarrow 9y = 6x \Rightarrow y = \frac{2}{3}x$.

(e) The tip of the shadow moves at a rate of $\frac{d}{dt}(x + y) = \frac{d}{dt}\left(x + \frac{2}{3}x\right) = \frac{5}{3} \frac{dx}{dt} = \frac{5}{3}(5) = \frac{25}{3}$ ft/s.

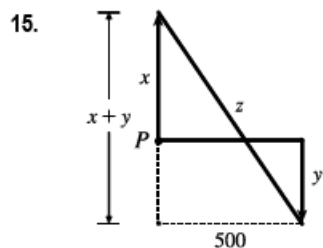


We are given that $\frac{dx}{dt} = 60$ mi/h and $\frac{dy}{dt} = 25$ mi/h. $z^2 = x^2 + y^2 \Rightarrow$

$$2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt} \Rightarrow z \frac{dz}{dt} = x \frac{dx}{dt} + y \frac{dy}{dt} \Rightarrow \frac{dz}{dt} = \frac{1}{z} \left(x \frac{dx}{dt} + y \frac{dy}{dt} \right).$$

After 2 hours, $x = 2(60) = 120$ and $y = 2(25) = 50 \Rightarrow z = \sqrt{120^2 + 50^2} = 130$,

$$\text{so } \frac{dz}{dt} = \frac{1}{z} \left(x \frac{dx}{dt} + y \frac{dy}{dt} \right) = \frac{120(60) + 50(25)}{130} = 65 \text{ mi/h.}$$



We are given that $\frac{dx}{dt} = 4$ ft/s and $\frac{dy}{dt} = 5$ ft/s. $z^2 = (x + y)^2 + 500^2 \Rightarrow$

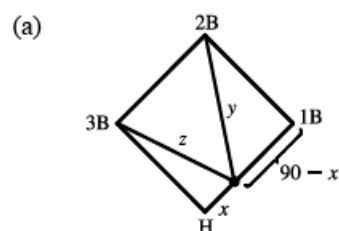
$$2z \frac{dz}{dt} = 2(x + y) \left(\frac{dx}{dt} + \frac{dy}{dt} \right). \text{ 15 minutes after the woman starts, we have}$$

$x = (4 \text{ ft/s})(20 \text{ min})(60 \text{ s/min}) = 4800$ ft and $y = 5 \cdot 15 \cdot 60 = 4500 \Rightarrow$

$z = \sqrt{(4800 + 4500)^2 + 500^2} = \sqrt{86,740,000}$, so

$$\frac{dz}{dt} = \frac{x + y}{z} \left(\frac{dx}{dt} + \frac{dy}{dt} \right) = \frac{4800 + 4500}{\sqrt{86,740,000}} (4 + 5) = \frac{837}{\sqrt{8674}} \approx 8.99 \text{ ft/s.}$$

16. We are given that $\frac{dx}{dt} = 24$ ft/s.



$y^2 = (90 - x)^2 + 90^2 \Rightarrow 2y \frac{dy}{dt} = 2(90 - x) \left(-\frac{dx}{dt} \right)$. When $x = 45$,

$$y = \sqrt{45^2 + 90^2} = 45\sqrt{5}, \text{ so } \frac{dy}{dt} = \frac{90 - x}{y} \left(-\frac{dx}{dt} \right) = \frac{45}{45\sqrt{5}} (-24) = -\frac{24}{\sqrt{5}},$$

so the distance from second base is decreasing at a rate of $\frac{24}{\sqrt{5}} \approx 10.7$ ft/s.

(b) Due to the symmetric nature of the problem in part (a), we expect to get the same answer—and we do.

$$z^2 = x^2 + 90^2 \Rightarrow 2z \frac{dz}{dt} = 2x \frac{dx}{dt}. \text{ When } x = 45, z = 45\sqrt{5}, \text{ so } \frac{dz}{dt} = \frac{45}{45\sqrt{5}} (24) = \frac{24}{\sqrt{5}} \approx 10.7 \text{ ft/s.}$$

17. $A = \frac{1}{2}bh$, where b is the base and h is the altitude. We are given that $\frac{dh}{dt} = 1$ cm/min and $\frac{dA}{dt} = 2$ cm²/min. Using the

Product Rule, we have $\frac{dA}{dt} = \frac{1}{2} \left(b \frac{dh}{dt} + h \frac{db}{dt} \right)$. When $h = 10$ and $A = 100$, we have $100 = \frac{1}{2}b(10) \Rightarrow \frac{1}{2}b = 10 \Rightarrow$

$$b = 20, \text{ so } 2 = \frac{1}{2} \left(20 \cdot 1 + 10 \frac{db}{dt} \right) \Rightarrow 4 = 20 + 10 \frac{db}{dt} \Rightarrow \frac{db}{dt} = \frac{4 - 20}{10} = -1.6 \text{ cm/min.}$$

26. We are given $dx/dt = 8$ ft/s. $\cot \theta = \frac{x}{100} \Rightarrow x = 100 \cot \theta \Rightarrow$

$$\frac{dx}{dt} = -100 \csc^2 \theta \frac{d\theta}{dt} \Rightarrow \frac{d\theta}{dt} = -\frac{\sin^2 \theta}{100} \cdot 8. \text{ When } y = 200, \sin \theta = \frac{100}{200} = \frac{1}{2} \Rightarrow$$

$$\frac{d\theta}{dt} = -\frac{(1/2)^2}{100} \cdot 8 = -\frac{1}{50} \text{ rad/s. The angle is decreasing at a rate of } \frac{1}{50} \text{ rad/s.}$$

