1. (a) $s = f(t) = t^3 - 12t^2 + 36t \quad \Rightarrow \quad v(t) = f'(t) = 3t^2 - 24t + 36$

(b) $v(3) = 27 - 72 + 36 = -9$ m/s

(c) The particle is at rest when $v(t) = 0$. $3t^2 - 24t + 36 = 0 \quad \Rightarrow \quad 3(t-2)(t-6) = 0 \quad \Rightarrow \quad t = 2, 6$.

(d) The particle is moving in the positive direction when $v(t) > 0$. $3(t-2)(t-6) > 0 \quad \Leftrightarrow \quad 0 \leq t < 2$ or $t > 6$.

(e) Since the particle is moving forward and backward, we need to calculate the distance traveled in the intervals $[0, 2]$, $[2, 6]$, and $[6, 8]$ separately.

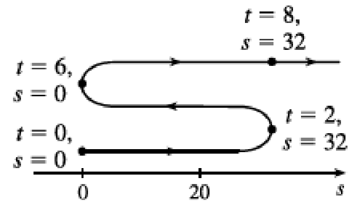
$$|f(2) - f(0)| = |32 - 0| = 32.$$

$$|f(6) - f(2)| = |0 - 32| = 32.$$

$$|f(8) - f(6)| = |32 - 0| = 32.$$

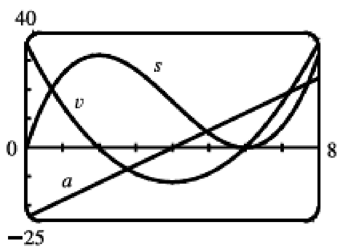
The total distance is $32 + 32 + 32 = 96$ m.

(f)



(g) $s = f(t) = t^3 - 12t^2 + 36t, t \geq 0 \quad \Rightarrow \quad v(t) = f'(t) = 3t^2 - 24t + 36$. $a(t) = v'(t) = 6t - 24$.
$a(3) = 6(3) - 24 = -6$ (m/s)/s or m/s^2.

(h)



(i) The particle is speeding up when v and a have the same sign. This occurs when $2 < t < 4$ and when $t > 6$. It is slowing down when v and a have opposite signs; that is, when $0 \leq t < 2$ and when $4 < t < 6$.

3. (a) From the figure, the velocity v is positive on the interval $(0, 2)$ and negative on the interval $(2, 3)$. The acceleration a is positive (negative) when the slope of the tangent line is positive (negative), so the acceleration is positive on the interval $(0, 1)$, and negative on the interval $(1, 3)$. The particle is speeding up when v and a have the same sign, that is, on the interval $(0, 1)$ when $v > 0$ and $a > 0$, and on the interval $(2, 3)$ when $v < 0$ and $a < 0$. The particle is slowing down when v and a have opposite signs, that is, on the interval $(1, 2)$ when $v > 0$ and $a < 0$.

(b) $v > 0$ on $(0, 3)$ and $v < 0$ on $(3, 4)$. $a > 0$ on $(1, 2)$ and $a < 0$ on $(0, 1)$ and $(2, 4)$. The particle is speeding up on $(1, 2)$ $[v > 0, a > 0]$ and on $(3, 4)$ $[v < 0, a < 0]$. The particle is slowing down on $(0, 1)$ and $(2, 3)$ $[v > 0, a < 0]$.

5. (a) $s(t) = t^3 - 4.5t^2 - 7t \quad \Rightarrow \quad v(t) = s'(t) = 3t^2 - 9t - 7 = 5 \quad \Leftrightarrow \quad 3t^2 - 9t - 12 = 0 \quad \Leftrightarrow$
$3(t-4)(t+1) = 0 \quad \Leftrightarrow \quad t = 4$ or -1. Since $t \geq 0$, the particle reaches a velocity of 5 m/s at $t = 4$ s.

(b) $a(t) = v'(t) = 6t - 9 = 0 \quad \Leftrightarrow \quad t = 1.5$. The acceleration changes from negative to positive, so the velocity changes from decreasing to increasing. Thus, at $t = 1.5$ s, the velocity has its minimum value.

6. (a) $s = 5t + 3t^2 \quad \Rightarrow \quad v(t) = \dfrac{ds}{dt} = 5 + 6t$, so $v(2) = 5 + 6(2) = 17$ m/s.

(b) $v(t) = 35 \quad \Rightarrow \quad 5 + 6t = 35 \quad \Rightarrow \quad 6t = 30 \quad \Rightarrow \quad t = 5$ s.

7. (a) $h = 10t - 0.83t^2 \quad \Rightarrow \quad v(t) = \dfrac{dh}{dt} = 10 - 1.66t$, so $v(3) = 10 - 1.66(3) = 5.02$ m/s.

(b) $h = 25 \quad \Rightarrow \quad 10t - 0.83t^2 = 25 \quad \Rightarrow \quad 0.83t^2 - 10t + 25 = 0 \quad \Rightarrow \quad t = \dfrac{10 \pm \sqrt{17}}{1.66} \approx 3.54$ or 8.51.

The value $t_1 = \left(10 - \sqrt{17}\right)/1.66$ corresponds to the time it takes for the stone to rise 25 m and $t_2 = \left(10 + \sqrt{17}\right)/1.66$ corresponds to the time when the stone is 25 m high on the way down. Thus, $v(t_1) = 10 - 1.66\left[\left(10 - \sqrt{17}\right)/1.66\right] = \sqrt{17} \approx 4.12$ m/s.

8. (a) At maximum height the velocity of the ball is 0 ft/s. $v(t) = s'(t) = 80 - 32t = 0 \quad \Leftrightarrow \quad 32t = 80 \quad \Leftrightarrow \quad t = \frac{5}{2}$.

So the maximum height is $s\left(\frac{5}{2}\right) = 80\left(\frac{5}{2}\right) - 16\left(\frac{5}{2}\right)^2 = 200 - 100 = 100$ ft.

(b) $s(t) = 80t - 16t^2 = 96 \quad \Leftrightarrow \quad 16t^2 - 80t + 96 = 0 \quad \Leftrightarrow \quad 16(t^2 - 5t + 6) = 0 \quad \Leftrightarrow \quad 16(t-3)(t-2) = 0$.

So the ball has a height of 96 ft on the way up at $t = 2$ and on the way down at $t = 3$. At these times the velocities are $v(2) = 80 - 32(2) = 16$ ft/s and $v(3) = 80 - 32(3) = -16$ ft/s, respectively.