1. 



The line from $P(2, f(2))$ to $Q(2+h, f(2+h))$ is the line that has slope $\frac{f(2+h)-f(2)}{h}$
3. $g^{\prime}(0)$ is the only negative value. The slope at $x=4$ is smaller than the slope at $x=2$ and both are smaller than the slope at $x=-2$. Thus, $g^{\prime}(0)<0<g^{\prime}(4)<g^{\prime}(2)<g^{\prime}(-2)$.
4. (a) Since $g(5)=-3$, the point $(5,-3)$ is on the graph of $g$. Since $g^{\prime}(5)=4$, the slope of the tangent line at $x=5$ is 4 .

Using the point-slope form of a line gives us $y-(-3)=4(x-5)$, or $y=4 x-23$.
(b) Since $(4,3)$ is on $y=f(x), f(4)=3$. The slope of the tangent line between $(0,2)$ and $(4,3)$ is $\frac{1}{4}$, so $f^{\prime}(4)=\frac{1}{4}$.
5. We begin by drawing a curve through the origin with a slope of 3 to satisfy $f(0)=0$ and $f^{\prime}(0)=3$. Since $f^{\prime}(1)=0$, we will round off our figure so that there is a horizontal tangent directly over $x=1$. Lastly, we make sure that the curve has a slope of -1 as we pass over $x=2$. Two of the many possibilities are shown.


7. Using Definition 2 with $f(x)=3 x^{2}-5 x$ and the point $(2,2)$, we have

$$
\begin{aligned}
f^{\prime}(2) & =\lim _{h \rightarrow 0} \frac{f(2+h)-f(2)}{h}=\lim _{h \rightarrow 0} \frac{\left[3(2+h)^{2}-5(2+h)\right]-2}{h} \\
& =\lim _{h \rightarrow 0} \frac{\left(12+12 h+3 h^{2}-10-5 h\right)-2}{h}=\lim _{h \rightarrow 0} \frac{3 h^{2}+7 h}{h}=\lim _{h \rightarrow 0}(3 h+7)=7
\end{aligned}
$$

So an equation of the tangent line at $(2,2)$ is $y-2=7(x-2)$ or $y=7 x-12$.
32. For 1910: We will average the difference quotients obtained using the years 1900 and 1920.

Let $A=\frac{E(1900)-E(1910)}{1900-1910}=\frac{48.3-51.1}{-10}=0.28$ and
$B=\frac{E(1920)-E(1910)}{1920-1910}=\frac{55.2-51.1}{10}=0.41$.
Then $E^{\prime}(1910)=\lim _{t \rightarrow 1910} \frac{E(t)-E(1910)}{t-1910} \approx \frac{A+B}{2}=0.345$. This means that life expectancy at birth was increasing at about 0.345 year/year in 1910 .

For 1950: Using data for 1940 and 1960 in a similar fashion, we obtain $E^{\prime}(1950) \approx[0.31+0.10] / 2=0.205$. So life expectancy at birth was increasing at about 0.205 year/year in 1950 .

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