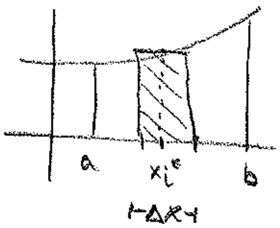
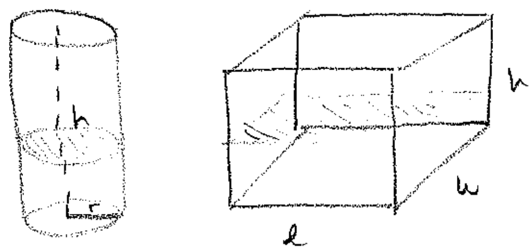
Recall

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} \overbrace{f(x_i^*)\,\Delta x}^{\text{area of rect.}} = \int_a^b f(x)\,dx$$



The key here was that we are adding <u>products</u>

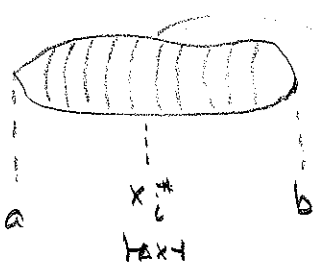
Transition to Volume ...



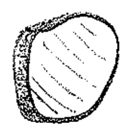
The volume of each of these figures is (area base) $\times$ height

But each one of these cross-sections are congruent
How do you find the Volume of an object that doesn't have congruent crossections?

<u>Loaf of Bread</u>



1) cut it into slices (equal width) Δx
2) pick a slice (x_i^*)



3) Volume of this slice
$V_i = a(x_i^*) \cdot \Delta x$

4) To find the approx Volume now add volume of all slices

$$\sum_{i=1}^{n} V(x_i^*) = \sum_{i=1}^{n} a(x_i^*) \cdot \Delta x$$

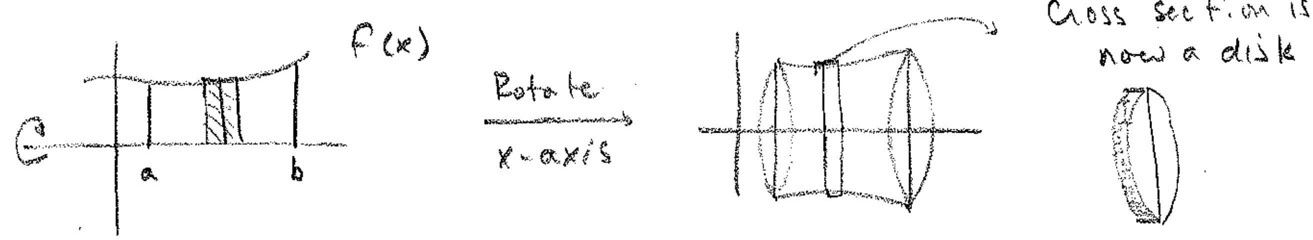
5) To find the exact Volume add up an infinite number of slices

$$V = \lim_{n\to\infty} \sum_{i=1}^{n} a(x_i^*) \cdot \Delta x = \int_a^b a(x)\,dx$$

where $a(x)$ is the area of each cross-section face.

Disc Method:

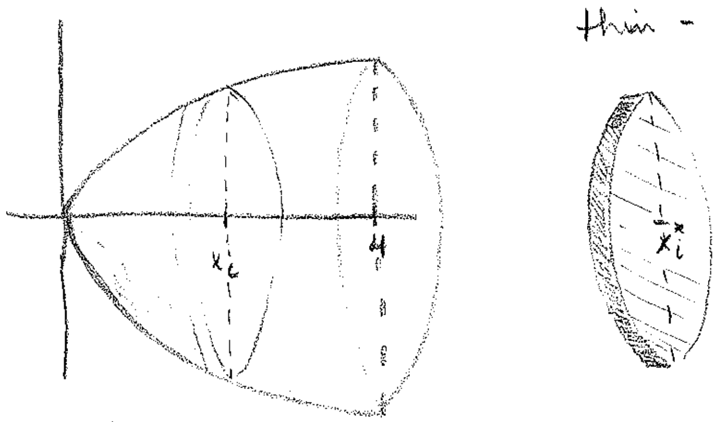
Some solids can be generated by rotating around x-axis, y-axis, or other lines.



whose cross sectional face is a circle.

<u>Example</u> Rotate the region bounded by $y = \sqrt{x}$ $x = 0$, $x = 4$ about the x-axis and find the volume of the resulting solid.

thin - Cross-section



area of face $= \pi (\sqrt{x})^2$

So Volume $= \int_0^4 \pi (\sqrt{x})^2 \, dx$

In General: Volume for Disc Method is $\int_a^b \pi (f(x))^2 \, dx$.