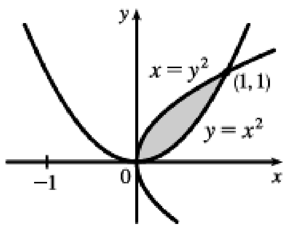
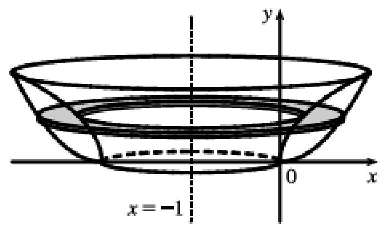
11. $y = x^2 \quad \Rightarrow \quad x = \sqrt{y}$ for $x \geq 0$. The outer radius is the distance from $x = -1$ to $x = \sqrt{y}$ and the inner radius is the distance from $x = -1$ to $x = y^2$.

$$V = \int_0^1 \pi \left\{ [\sqrt{y} - (-1)]^2 - \left[y^2 - (-1)\right]^2 \right\} dy = \pi \int_0^1 \left[(\sqrt{y} + 1)^2 - \left(y^2 + 1\right)^2 \right] dy$$

$$= \pi \int_0^1 \left(y + 2\sqrt{y} + 1 - y^4 - 2y^2 - 1\right) dy = \pi \int_0^1 \left(y + 2\sqrt{y} - y^4 - 2y^2\right) dy$$

$$= \pi \left[\tfrac{1}{2}y^2 + \tfrac{4}{3}y^{3/2} - \tfrac{1}{5}y^5 - \tfrac{2}{3}y^3 \right]_0^1 = \pi\left(\tfrac{1}{2} + \tfrac{4}{3} - \tfrac{1}{5} - \tfrac{2}{3}\right) = \tfrac{29}{30}\pi$$





6. (a) $f_{\text{ave}} = \frac{1}{3-1} \int_1^3 \ln x \, dx = \frac{1}{2}[x \ln x - x]_1^3$ [by parts]

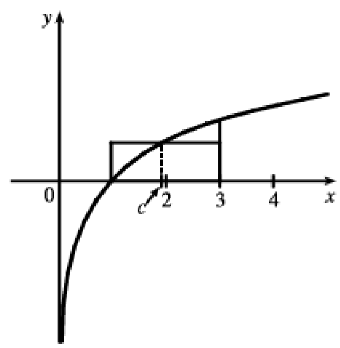
$= \frac{1}{2}[(3 \ln 3 - 3) - (\ln 1 - 1)]$

$= \frac{1}{2}(3 \ln 3 - 2) = \frac{3}{2} \ln 3 - 1$

(b) $f_{\text{ave}} = f(c) \quad \Leftrightarrow \quad \frac{3}{2} \ln 3 - 1 = \ln c \quad \Leftrightarrow$

$c = e^{(3/2)\ln 3 - 1}$ or $c = 3\sqrt{3}/e \approx 1.91$

(c)



10. The requirement is that $\dfrac{1}{b-0} \displaystyle\int_0^b f(x)\, dx = 3$. The LHS of this equation is equal to

$\dfrac{1}{b} \displaystyle\int_0^b \left(2 + 6x - 3x^2\right) dx = \dfrac{1}{b}\left[2x + 3x^2 - x^3\right]_0^b = 2 + 3b - b^2$, so we solve the equation $2 + 3b - b^2 = 3 \quad \Leftrightarrow$

$b^2 - 3b + 1 = 0 \quad \Leftrightarrow \quad b = \dfrac{3 \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \dfrac{3 \pm \sqrt{5}}{2}$. Both roots are valid since they are positive.