

$$1. A = \int_{x=0}^{x=4} (y_T - y_B) dx = \int_0^4 [(5x - x^2) - x] dx = \int_0^4 (4x - x^2) dx$$

$$= [2x^2 - \frac{1}{3}x^3]_0^4 = (32 - \frac{64}{3}) - (0) = \frac{32}{3}$$

$$2. A = \int_0^2 \left(\sqrt{x+2} - \frac{1}{x+1} \right) dx = \left[\frac{2}{3}(x+2)^{3/2} - \ln(x+1) \right]_0^2$$

$$= \left[\frac{2}{3}(4)^{3/2} - \ln 3 \right] - \left[\frac{2}{3}(2)^{3/2} - \ln 1 \right] = \frac{16}{3} - \ln 3 - \frac{4}{3}\sqrt{2}$$

9. $12 - x^2 = x^2 - 6 \Leftrightarrow 2x^2 = 18 \Leftrightarrow x^2 = 9 \Leftrightarrow x = \pm 3$, so

$$A = \int_{-3}^3 [(12 - x^2) - (x^2 - 6)] dx = 2 \int_0^3 (18 - 2x^2) dx \quad \text{[by symmetry]}$$

$$= 2 \left[18x - \frac{2}{3}x^3 \right]_0^3 = 2 [(54 - 18) - 0] = 2(36) = 72$$

