

15. Let $u = 3ax + bx^3$. Then $du = (3a + 3bx^2) dx = 3(a + bx^2) dx$, so

$$\int \frac{a + bx^2}{\sqrt{3ax + bx^3}} dx = \int \frac{\frac{1}{3} du}{u^{1/2}} = \frac{1}{3} \int u^{-1/2} du = \frac{1}{3} \cdot 2u^{1/2} + C = \frac{2}{3} \sqrt{3ax + bx^3} + C.$$

21. Let $u = \sin \theta$. Then $du = \cos \theta d\theta$, so $\int \cos \theta \sin^6 \theta d\theta = \int u^6 du = \frac{1}{7} u^7 + C = \frac{1}{7} \sin^7 \theta + C$.

29. $\int \frac{e^x + 1}{e^x} dx = \int \left(\frac{e^x}{e^x} + \frac{1}{e^x} \right) dx = \int (1 + e^{-x}) dx = x - e^{-x} + C$ [Substitute $u = -x$.]

59. **First Figure** Let $u = \sqrt{x}$, so $x = u^2$ and $dx = 2u du$. When $x = 0$, $u = 0$; when $x = 1$, $u = 1$. Thus,

$$A_1 = \int_0^1 e^{\sqrt{x}} dx = \int_0^1 e^u (2u du) = 2 \int_0^1 u e^u du.$$

Second Figure $A_2 = \int_0^1 2x e^x dx = 2 \int_0^1 u e^u du$.

Third Figure Let $u = \sin x$, so $du = \cos x dx$. When $x = 0$, $u = 0$; when $x = \frac{\pi}{2}$, $u = 1$. Thus,

$$A_3 = \int_0^{\pi/2} e^{\sin x} \sin 2x dx = \int_0^{\pi/2} e^{\sin x} (2 \sin x \cos x) dx = \int_0^1 e^u (2u du) = 2 \int_0^1 u e^u du.$$

Since $A_1 = A_2 = A_3$, all three areas are equal.

63. Let $u = 2x$. Then $du = 2 dx$, so $\int_0^2 f(2x) dx = \int_0^4 f(u) \left(\frac{1}{2} du \right) = \frac{1}{2} \int_0^4 f(u) du = \frac{1}{2}(10) = 5$.