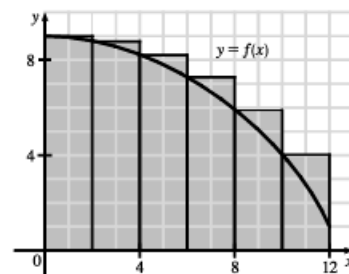
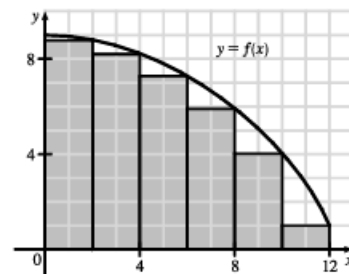


$$\begin{aligned}
 2. (a) (i) L_6 &= \sum_{i=1}^6 f(x_{i-1}) \Delta x \quad [\Delta x = \frac{12-0}{6} = 2] \\
 &= 2[f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)] \\
 &= 2[f(0) + f(2) + f(4) + f(6) + f(8) + f(10)] \\
 &\approx 2(9 + 8.8 + 8.2 + 7.3 + 5.9 + 4.1) \\
 &= 2(43.3) = 86.6
 \end{aligned}$$

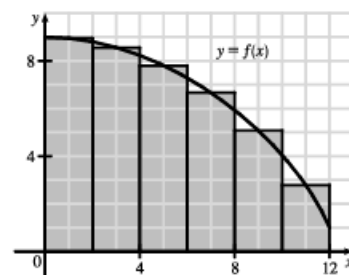


$$\begin{aligned}
 (ii) R_6 &= L_6 + 2 \cdot f(12) - 2 \cdot f(0) \\
 &\approx 86.6 + 2(1) - 2(9) = 70.6
 \end{aligned}$$

[Add area of rightmost lower rectangle and subtract area of leftmost upper rectangle.]



$$\begin{aligned}
 (iii) M_6 &= \sum_{i=1}^6 f(x_i^*) \Delta x \\
 &= 2[f(1) + f(3) + f(5) + f(7) + f(9) + f(11)] \\
 &\approx 2(8.9 + 8.5 + 7.8 + 6.6 + 5.1 + 2.8) \\
 &= 2(39.7) = 79.4
 \end{aligned}$$



(b) Since f is decreasing, we obtain an *overestimate* by using *left* endpoints; that is, L_6 .

(c) Since f is decreasing, we obtain an *underestimate* by using *right* endpoints; that is, R_6 .

(d) M_6 gives the best estimate, since the area of each rectangle appears to be closer to the true area than the overestimates and underestimates in L_6 and R_6 .

$$5. (a) f(x) = 1 + x^2 \text{ and } \Delta x = \frac{2 - (-1)}{3} = 1 \Rightarrow$$

$$R_3 = 1 \cdot f(0) + 1 \cdot f(1) + 1 \cdot f(2) = 1 \cdot 1 + 1 \cdot 2 + 1 \cdot 5 = 8.$$

$$\Delta x = \frac{2 - (-1)}{6} = 0.5 \Rightarrow$$

$$\begin{aligned} R_6 &= 0.5[f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5) + f(2)] \\ &= 0.5(1.25 + 1 + 1.25 + 2 + 3.25 + 5) \\ &= 0.5(13.75) = 6.875 \end{aligned}$$

$$(b) L_3 = 1 \cdot f(-1) + 1 \cdot f(0) + 1 \cdot f(1) = 1 \cdot 2 + 1 \cdot 1 + 1 \cdot 2 = 5$$

$$\begin{aligned} L_6 &= 0.5[f(-1) + f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5)] \\ &= 0.5(2 + 1.25 + 1 + 1.25 + 2 + 3.25) \\ &= 0.5(10.75) = 5.375 \end{aligned}$$

$$(c) M_3 = 1 \cdot f(-0.5) + 1 \cdot f(0.5) + 1 \cdot f(1.5)$$

$$= 1 \cdot 1.25 + 1 \cdot 1.25 + 1 \cdot 3.25 = 5.75$$

$$\begin{aligned} M_6 &= 0.5[f(-0.75) + f(-0.25) + f(0.25) \\ &\quad + f(0.75) + f(1.25) + f(1.75)] \\ &= 0.5(1.5625 + 1.0625 + 1.0625 + 1.5625 + 2.5625 + 4.0625) \\ &= 0.5(11.875) = 5.9375 \end{aligned}$$

(d) M_6 appears to be the best estimate.

11. Since v is an increasing function, L_6 will give us a lower estimate and R_6 will give us an upper estimate.

$$\begin{aligned} L_6 &= (0 \text{ ft/s})(0.5 \text{ s}) + (6.2)(0.5) + (10.8)(0.5) + (14.9)(0.5) + (18.1)(0.5) + (19.4)(0.5) \\ &= 0.5(69.4) = 34.7 \text{ ft} \end{aligned}$$

$$R_6 = 0.5(6.2 + 10.8 + 14.9 + 18.1 + 19.4 + 20.2) = 0.5(89.6) = 44.8 \text{ ft}$$

15. For a decreasing function, using left endpoints gives us an overestimate and using right endpoints results in an underestimate.

We will use M_6 to get an estimate. $\Delta t = 1$, so

$$\begin{aligned} M_6 &= 1[v(0.5) + v(1.5) + v(2.5) + v(3.5) + v(4.5) + v(5.5)] \\ &\approx 55 + 40 + 28 + 18 + 10 + 4 = 155 \text{ ft} \end{aligned}$$

For a very rough check on the above calculation, we can draw a line from $(0, 70)$ to $(6, 0)$ and calculate the area of the triangle: $\frac{1}{2}(70)(6) = 210$. This is clearly an overestimate, so our midpoint estimate of 155 is reasonable.

