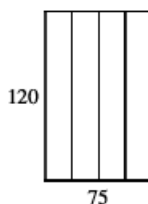
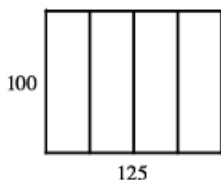


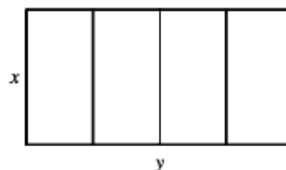
7. (a)



The areas of the three figures are 12,500, 12,500, and 9000 ft². There appears to be a maximum area of at least 12,500 ft².

(b) Let x denote the length of each of two sides and three dividers.

Let y denote the length of the other two sides.



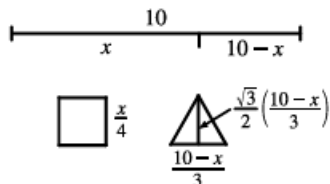
(c) Area $A = \text{length} \times \text{width} = y \cdot x$

(d) Length of fencing = 750 $\Rightarrow 5x + 2y = 750$

(e) $5x + 2y = 750 \Rightarrow y = 375 - \frac{5}{2}x \Rightarrow A(x) = (375 - \frac{5}{2}x)x = 375x - \frac{5}{2}x^2$

(f) $A'(x) = 375 - 5x = 0 \Rightarrow x = 75$. Since $A''(x) = -5 < 0$ there is an absolute maximum when $x = 75$. Then $y = \frac{375}{2} = 187.5$. The largest area is $75(\frac{375}{2}) = 14,062.5$ ft². These values of x and y are between the values in the first and second figures in part (a). Our original estimate was low.

21.



Let x be the length of the wire used for the square. The total area is

$$A(x) = \left(\frac{x}{4}\right)^2 + \frac{1}{2} \left(\frac{10-x}{3}\right) \frac{\sqrt{3}}{2} \left(\frac{10-x}{3}\right)$$

$$= \frac{1}{16}x^2 + \frac{\sqrt{3}}{36}(10-x)^2, \quad 0 \leq x \leq 10$$

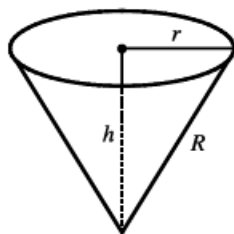
$$A'(x) = \frac{1}{8}x - \frac{\sqrt{3}}{18}(10-x) = 0 \Leftrightarrow \frac{9}{72}x + \frac{4\sqrt{3}}{72}x - \frac{40\sqrt{3}}{72} = 0 \Leftrightarrow x = \frac{40\sqrt{3}}{9+4\sqrt{3}}. \text{ Now } A(0) = \left(\frac{\sqrt{3}}{36}\right)100 \approx 4.81,$$

$$A(10) = \frac{100}{16} = 6.25 \text{ and } A\left(\frac{40\sqrt{3}}{9+4\sqrt{3}}\right) \approx 2.72, \text{ so}$$

(a) The maximum area occurs when $x = 10$ m, and all the wire is used for the square.

(b) The minimum area occurs when $x = \frac{40\sqrt{3}}{9+4\sqrt{3}} \approx 4.35$ m.

23.



$$h^2 + r^2 = R^2 \Rightarrow V = \frac{\pi}{3}r^2h = \frac{\pi}{3}(R^2 - h^2)h = \frac{\pi}{3}(R^2h - h^3).$$

$$V'(h) = \frac{\pi}{3}(R^2 - 3h^2) = 0 \text{ when } h = \frac{1}{\sqrt{3}}R. \text{ This gives an absolute}$$

maximum, since $V'(h) > 0$ for $0 < h < \frac{1}{\sqrt{3}}R$ and $V'(h) < 0$ for

$h > \frac{1}{\sqrt{3}}R$. The maximum volume is

$$V\left(\frac{1}{\sqrt{3}}R\right) = \frac{\pi}{3}\left(\frac{1}{\sqrt{3}}R^3 - \frac{1}{3\sqrt{3}}R^3\right) = \frac{2}{9\sqrt{3}}\pi R^3.$$