39. $2x^2 + y^2 = 3$ and $x = y^2$ intersect when $2x^2 + x - 3 = 0 \quad \Leftrightarrow \quad (2x+3)(x-1) = 0 \quad \Leftrightarrow \quad x = -\frac{3}{2}$ or 1, but $-\frac{3}{2}$ is extraneous since $x = y^2$ is nonnegative. When $x = 1$, $1 = y^2 \quad \Rightarrow \quad y = \pm 1$, so there are two points of intersection: $(1, \pm 1)$. $2x^2 + y^2 = 3 \quad \Rightarrow \quad 4x + 2yy' = 0 \quad \Rightarrow \quad y' = -2x/y$, and $x = y^2 \quad \Rightarrow \quad 1 = 2yy' \quad \Rightarrow \quad y' = 1/(2y)$. At $(1, 1)$, the slopes are $m_1 = -2(1)/1 = -2$ and $m_2 = 1/(2 \cdot 1) = \frac{1}{2}$, so the curves are orthogonal (since m_1 and m_2 are negative reciprocals of each other). By symmetry, the curves are also orthogonal at $(1, -1)$.

3. $f(\theta) = \ln(\cos\theta) \quad \Rightarrow \quad f'(\theta) = \dfrac{1}{\cos\theta} \dfrac{d}{d\theta}(\cos\theta) = \dfrac{-\sin\theta}{\cos\theta} = -\tan\theta$

4. $f(x) = \cos(\ln x) \quad \Rightarrow \quad f'(x) = -\sin(\ln x) \cdot \dfrac{1}{x} = \dfrac{-\sin(\ln x)}{x}$

5. $f(x) = \sqrt[5]{\ln x} = (\ln x)^{1/5} \quad \Rightarrow \quad f'(x) = \frac{1}{5}(\ln x)^{-4/5} \dfrac{d}{dx}(\ln x) = \dfrac{1}{5(\ln x)^{4/5}} \cdot \dfrac{1}{x} = \dfrac{1}{5x\,\sqrt[5]{(\ln x)^4}}$

6. $f(x) = \ln \sqrt[5]{x} = \ln x^{1/5} = \frac{1}{5}\ln x \quad \Rightarrow \quad f'(x) = \dfrac{1}{5} \cdot \dfrac{1}{x} = \dfrac{1}{5x}$

7. $f(x) = \log_2(1 - 3x) \quad \Rightarrow \quad f'(x) = \dfrac{1}{(1-3x)\ln 2} \dfrac{d}{dx}(1 - 3x) = \dfrac{-3}{(1-3x)\ln 2}$ or $\dfrac{3}{(3x-1)\ln 2}$

8. $f(x) = \log_5(xe^x) \quad \Rightarrow \quad f'(x) = \dfrac{1}{xe^x \ln 5} \dfrac{d}{dx}(xe^x) = \dfrac{1}{xe^x \ln 5}(xe^x + e^x \cdot 1) = \dfrac{e^x(x+1)}{xe^x \ln 5} = \dfrac{x+1}{x \ln 5}$

Another solution: We can change the form of the function by first using logarithm properties.

$f(x) = \log_5(xe^x) = \log_5 x + \log_5 e^x \quad \Rightarrow \quad f'(x) = \dfrac{1}{x\ln 5} + \dfrac{1}{e^x \ln 5} \cdot e^x = \dfrac{1}{x \ln 5} + \dfrac{1}{\ln 5}$ or $\dfrac{1+x}{x\ln 5}$

23. $y = \ln(x^2 - 3) \quad \Rightarrow \quad y' = \dfrac{1}{x^2 - 3} \cdot 2x = \dfrac{2x}{x^2 - 3}$.

$y'(2) = \dfrac{2(2)}{2^2 - 3} = 4$, so an equation of the tangent line at $(2, 0)$ is $y - 0 = 4(x - 2)$ or $y = 4x - 8$.

27. $y = (2x+1)^5(x^4-3)^6 \quad \Rightarrow \quad \ln y = \ln\big((2x+1)^5(x^4-3)^6\big) \quad \Rightarrow \quad \ln y = 5\ln(2x+1) + 6\ln(x^4 - 3) \quad \Rightarrow$

$\dfrac{1}{y} y' = 5 \cdot \dfrac{1}{2x+1} \cdot 2 + 6 \cdot \dfrac{1}{x^4 - 3} \cdot 4x^3 \quad \Rightarrow$

$y' = y\left(\dfrac{10}{2x+1} + \dfrac{24x^3}{x^4-3}\right) = (2x+1)^5(x^4-3)^6\left(\dfrac{10}{2x+1} + \dfrac{24x^3}{x^4-3}\right)$.

[The answer could be simplified to $y' = 2(2x+1)^4(x^4-3)^5(29x^4 + 12x^3 - 15)$, but this is unnecessary.]

32. $y = x^{\cos x} \quad \Rightarrow \quad \ln y = \ln x^{\cos x} \quad \Rightarrow \quad \ln y = \cos x \ln x \quad \Rightarrow \quad \dfrac{1}{y} y' = \cos x \cdot \dfrac{1}{x} + \ln x \cdot (-\sin x) \quad \Rightarrow$

$y' = y\left(\dfrac{\cos x}{x} - \ln x \sin x\right) \quad \Rightarrow \quad y' = x^{\cos x}\left(\dfrac{\cos x}{x} - \ln x \sin x\right)$