5. $\dfrac{d}{dx}(x^2y+xy^2)=\dfrac{d}{dx}(3x) \;\Rightarrow\; (x^2y'+y\cdot 2x)+(x\cdot 2yy'+y^2\cdot 1)=3 \;\Rightarrow\; x^2y'+2xyy'=3-2xy-y^2 \;\Rightarrow$

$y'(x^2+2xy)=3-2xy-y^2 \;\Rightarrow\; y'=\dfrac{3-2xy-y^2}{x^2+2xy}$

7. $\dfrac{d}{dx}(x^2y^2+x\sin y)=\dfrac{d}{dx}(4) \;\Rightarrow\; x^2\cdot 2yy'+y^2\cdot 2x+x\cos y\cdot y'+\sin y\cdot 1=0 \;\Rightarrow$

$2x^2yy'+x\cos y\cdot y'=-2xy^2-\sin y \;\Rightarrow\; (2x^2y+x\cos y)y'=-2xy^2-\sin y \;\Rightarrow\; y'=\dfrac{-2xy^2-\sin y}{2x^2y+x\cos y}$

15. $x^2+xy+y^2=3 \;\Rightarrow\; 2x+xy'+y\cdot 1+2yy'=0 \;\Rightarrow\; xy'+2yy'=-2x-y \;\Rightarrow\; y'(x+2y)=-2x-y \;\Rightarrow$

$y'=\dfrac{-2x-y}{x+2y}$. When $x=1$ and $y=1$, we have $y'=\dfrac{-2-1}{1+2\cdot 1}=\dfrac{-3}{3}=-1$, so an equation of the tangent line is $y-1=-1(x-1)$ or $y=-x+2$.

22. (a) $y^2=x^3+3x^2 \;\Rightarrow\; 2yy'=3x^2+3(2x) \;\Rightarrow\; y'=\dfrac{3x^2+6x}{2y}$. So at the point $(1,-2)$ we have

$y'=\dfrac{3(1)^2+6(1)}{2(-2)}=-\dfrac{9}{4}$, and an equation of the tangent line is $y+2=-\frac{9}{4}(x-1)$ or $y=-\frac{9}{4}x+\frac{1}{4}$.

(b) The curve has a horizontal tangent where $y'=0 \;\Leftrightarrow$

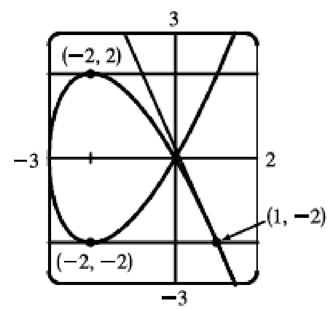
$3x^2+6x=0 \;\Leftrightarrow\; 3x(x+2)=0 \;\Leftrightarrow\; x=0$ or $x=-2$.

But note that at $x=0$, $y=0$ also, so the derivative does not exist.

At $x=-2$, $y^2=(-2)^3+3(-2)^2=-8+12=4$, so $y=\pm 2$.

So the two points at which the curve has a horizontal tangent are $(-2,-2)$ and $(-2,2)$.

(c)



25. From Exercise 19, a tangent to the lemniscate will be horizontal if $y'=0 \;\Rightarrow\; 25x-4x(x^2+y^2)=0 \;\Rightarrow$ $x[25-4(x^2+y^2)]=0 \;\Rightarrow\; x^2+y^2=\frac{25}{4}$ (1). (Note that when x is 0, y is also 0, and there is no horizontal tangent at the origin.) Substituting $\frac{25}{4}$ for x^2+y^2 in the equation of the lemniscate, $2(x^2+y^2)^2=25(x^2-y^2)$, we get $x^2-y^2=\frac{25}{8}$ (2). Solving (1) and (2), we have $x^2=\frac{75}{16}$ and $y^2=\frac{25}{16}$, so the four points are $\left(\pm\frac{5\sqrt{3}}{4},\pm\frac{5}{4}\right)$.

31. $y=\sin^{-1}(2x+1) \;\Rightarrow$

$y'=\dfrac{1}{\sqrt{1-(2x+1)^2}}\cdot\dfrac{d}{dx}(2x+1)=\dfrac{1}{\sqrt{1-(4x^2+4x+1)}}\cdot 2=\dfrac{2}{\sqrt{-4x^2-4x}}=\dfrac{1}{\sqrt{-x^2-x}}$

36. (a) Let $y=\cos^{-1}x$. Then $\cos y=x$ and $0\le y\le\pi \;\Rightarrow\; \dfrac{d}{dx}(\cos y)=\dfrac{d}{dx}(x) \;\Rightarrow\; -\sin y\,\dfrac{dy}{dx}=1 \;\Rightarrow$

$\dfrac{dy}{dx}=-\dfrac{1}{\sin y}=-\dfrac{1}{\sqrt{1-\cos^2 y}}=-\dfrac{1}{\sqrt{1-x^2}}$ (Note that $\sin y\ge 0$ for $0\le y\le\pi$.)

(b) $y=x\cos^{-1}x-\sqrt{1-x^2} \;\Rightarrow\; y'=\cos^{-1}x-\dfrac{x}{\sqrt{1-x^2}}+\dfrac{x}{\sqrt{1-x^2}}=\cos^{-1}x$