

$$3. y = \sqrt{x} + \frac{1}{\sqrt[3]{x^4}} = x^{1/2} + x^{-4/3} \Rightarrow y' = \frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-7/3} = \frac{1}{2\sqrt{x}} - \frac{4}{3\sqrt[3]{x^7}}$$

$$6. y = \frac{e^x}{1+x^2} \Rightarrow y' = \frac{(1+x^2)e^x - e^x(2x)}{(1+x^2)^2} = \frac{e^x(x^2 - 2x + 1)}{(1+x^2)^2} = \frac{e^x(x-1)^2}{(1+x^2)^2}$$

$$8. y = e^{-t}(t^2 - 2t + 2) \Rightarrow y' = e^{-t}(2t - 2) + (t^2 - 2t + 2)(-e^{-t}) = e^{-t}(2t - 2 - t^2 + 2t - 2) = e^{-t}(-t^2 + 4t - 4)$$

$$9. y = \frac{t}{1-t^2} \Rightarrow y' = \frac{(1-t^2)(1) - t(-2t)}{(1-t^2)^2} = \frac{1-t^2+2t^2}{(1-t^2)^2} = \frac{t^2+1}{(1-t^2)^2}$$

39. $y = 4 \sin^2 x \Rightarrow y' = 4 \cdot 2 \sin x \cos x$. At $(\frac{\pi}{6}, 1)$, $y' = 8 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3}$, so an equation of the tangent line is $y - 1 = 2\sqrt{3}(x - \frac{\pi}{6})$, or $y = 2\sqrt{3}x + 1 - \pi\sqrt{3}/3$.

$$40. y = \frac{x^2 - 1}{x^2 + 1} \Rightarrow y' = \frac{(x^2 + 1)(2x) - (x^2 - 1)(2x)}{(x^2 + 1)^2} = \frac{4x}{(x^2 + 1)^2}$$

At $(0, -1)$, $y' = 0$, so an equation of the tangent line is $y + 1 = 0(x - 0)$, or $y = -1$.

$$41. y = (2+x)e^{-x} \Rightarrow y' = (2+x)(-e^{-x}) + e^{-x} \cdot 1 = e^{-x}[-(2+x) + 1] = e^{-x}(-x-1)$$

At $(0, 2)$, $y' = 1(-1) = -1$, so an equation of the tangent line is $y - 2 = -1(x - 0)$, or $y = -x + 2$.

The slope of the normal line is 1, so an equation of the normal line is $y - 2 = 1(x - 0)$, or $y = x + 2$.

$$47. (a) h(x) = f(x)g(x) \Rightarrow h'(x) = f(x)g'(x) + g(x)f'(x) \Rightarrow h'(2) = f(2)g'(2) + g(2)f'(2) = (3)(4) + (5)(-2) = 12 - 10 = 2$$

$$(b) F(x) = f(g(x)) \Rightarrow F'(x) = f'(g(x))g'(x) \Rightarrow F'(2) = f'(g(2))g'(2) = f'(5)(4) = 11 \cdot 4 = 44$$

$$48. (a) P(x) = f(x)g(x) \Rightarrow P'(x) = f(x)g'(x) + g(x)f'(x) \Rightarrow$$

$$P'(2) = f(2)g'(2) + g(2)f'(2) = (1)\left(\frac{6-0}{3-0}\right) + (4)\left(\frac{0-3}{3-0}\right) = (1)(2) + (4)(-1) = 2 - 4 = -2$$

$$(b) Q(x) = \frac{f(x)}{g(x)} \Rightarrow Q'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \Rightarrow$$

$$Q'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2} = \frac{(4)(-1) - (1)(2)}{4^2} = \frac{-6}{16} = -\frac{3}{8}$$

$$(c) C(x) = f(g(x)) \Rightarrow C'(x) = f'(g(x))g'(x) \Rightarrow$$

$$C'(2) = f'(g(2))g'(2) = f'(4)g'(2) = \left(\frac{6-0}{5-3}\right)(2) = (3)(2) = 6$$

$$49. f(x) = x^2g(x) \Rightarrow f'(x) = x^2g'(x) + g(x)(2x) = x[xg'(x) + 2g(x)]$$

$$62. (a) f(x) = \frac{\ln x}{x} \Rightarrow f'(x) = \frac{x(1/x) - (\ln x) \cdot 1}{x^2} = \frac{1 - \ln x}{x^2}. f'(x) > 0 \Rightarrow 1 - \ln x > 0 \Rightarrow \ln x < 1 \Rightarrow x < e. \text{ Since the domain of } f \text{ is } (0, \infty), f \text{ is increasing on } (0, e).$$

$$(b) f''(x) = \frac{x^2(-1/x) - (1 - \ln x)(2x)}{(x^2)^2} = \frac{x[-1 - 2(1 - \ln x)]}{x^4} = \frac{2 \ln x - 3}{x^3}.$$

$$f''(x) > 0 \Rightarrow 2 \ln x - 3 > 0 \Rightarrow \ln x > \frac{3}{2} \Rightarrow x > e^{3/2} \approx 4.48. f \text{ is concave upward on } (e^{3/2}, \infty).$$

63. $y = f(x) = ax^2 + bx + c \Rightarrow f'(x) = 2ax + b$. We know that $f'(-1) = 6$ and $f'(5) = -2$, so $-2a + b = 6$ and $10a + b = -2$. Subtracting the first equation from the second gives $12a = -8 \Rightarrow a = -\frac{2}{3}$. Substituting $-\frac{2}{3}$ for a in the first equation gives $b = \frac{14}{3}$. Now $f(1) = 4 \Rightarrow 4 = a + b + c$, so $c = 4 + \frac{2}{3} - \frac{14}{3} = 0$ and hence, $f(x) = -\frac{2}{3}x^2 + \frac{14}{3}x$.

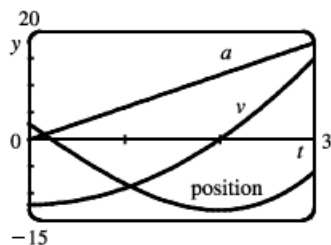
64. (a) $y = t^3 - 12t + 3 \Rightarrow v(t) = y' = 3t^2 - 12 \Rightarrow a(t) = v'(t) = 6t$

(b) $v(t) = 3(t^2 - 4) > 0$ when $t > 2$, so it moves upward when $t > 2$ and downward when $0 \leq t < 2$.

(c) Distance upward = $y(3) - y(2) = -6 - (-13) = 7$,

Distance downward = $y(0) - y(2) = 3 - (-13) = 16$. Total distance = $7 + 16 = 23$.

(d)



(e) The particle is speeding up when v and a have the same sign, that is, when $t > 2$. The particle is slowing down when v and a have opposite signs; that is, when $0 < t < 2$.