6. By the Quotient Rule, $y=\frac{e^{x}}{1+x} \Rightarrow y^{\prime}=\frac{(1+x) e^{x}-e^{x}(1)}{(1+x)^{2}}=\frac{e^{x}+x e^{x}-e^{x}}{(x+1)^{2}}=\frac{x e^{x}}{(x+1)^{2}}$.
7. $f(x)=\frac{A}{B+C e^{x}} \stackrel{\mathrm{QR}}{\Rightarrow} \quad f^{\prime}(x)=\frac{\left(B+C e^{x}\right) \cdot 0-A\left(C e^{x}\right)}{\left(B+C e^{x}\right)^{2}}=-\frac{A C e^{x}}{\left(B+C e^{x}\right)^{2}}$
8. $y=\frac{\sqrt{x}}{x+1} \Rightarrow y^{\prime}=\frac{(x+1)\left(\frac{1}{2 \sqrt{x}}\right)-\sqrt{x}(1)}{(x+1)^{2}}=\frac{(x+1)-(2 x)}{2 \sqrt{x}(x+1)^{2}}=\frac{1-x}{2 \sqrt{x}(x+1)^{2}}$. At $(4,0.4), y^{\prime}=\frac{-3}{100}=-0.03$, and an equation of the tangent line is $y-0.4=-0.03(x-4)$, or $y=-0.03 x+0.52$. The slope of the normal line is $\frac{100}{3}$, so an equation of the normal line is $y-0.4=\frac{100}{3}(x-4) \Leftrightarrow y=\frac{100}{3} x-\frac{400}{3}+\frac{2}{5} \Leftrightarrow y=\frac{100}{3} x-\frac{1994}{15}$.
9. (a) $y=f(x)=\frac{x}{1+x^{2}} \Rightarrow$
(b)

10. We are given that $f(5)=1, f^{\prime}(5)=6, g(5)=-3$, and $g^{\prime}(5)=2$.
(a) $(f g)^{\prime}(5)=f(5) g^{\prime}(5)+g(5) f^{\prime}(5)=(1)(2)+(-3)(6)=2-18=-16$
(b) $\left(\frac{f}{g}\right)^{\prime}(5)=\frac{g(5) f^{\prime}(5)-f(5) g^{\prime}(5)}{[g(5)]^{2}}=\frac{(-3)(6)-(1)(2)}{(-3)^{2}}=-\frac{20}{9}$
(c) $\left(\frac{g}{f}\right)^{\prime}(5)=\frac{f(5) g^{\prime}(5)-g(5) f^{\prime}(5)}{[f(5)]^{2}}=\frac{(1)(2)-(-3)(6)}{(1)^{2}}=20$
11. We are given that $f(3)=4, g(3)=2, f^{\prime}(3)=-6$, and $g^{\prime}(3)=5$.
(a) $(f+g)^{\prime}(3)=f^{\prime}(3)+g^{\prime}(3)=-6+5=-1$
(b) $(f g)^{\prime}(3)=f(3) g^{\prime}(3)+g(3) f^{\prime}(3)=(4)(5)+(2)(-6)=20-12=8$
(c) $\left(\frac{f}{g}\right)^{\prime}(3)=\frac{g(3) f^{\prime}(3)-f(3) g^{\prime}(3)}{[g(3)]^{2}}=\frac{(2)(-6)-(4)(5)}{(2)^{2}}=\frac{-32}{4}=-8$
(d) $\left(\frac{f}{f-g}\right)^{\prime}(3)=\frac{[f(3)-g(3)] f^{\prime}(3)-f(3)\left[f^{\prime}(3)-g^{\prime}(3)\right]}{[f(3)-g(3)]^{2}}=\frac{(4-2)(-6)-4(-6-5)}{(4-2)^{2}}=\frac{-12+44}{2^{2}}=8$
12. (a) From the graphs of $f$ and $g$, we obtain the following values: $f(1)=2$ since the point $(1,2)$ is on the graph of $f$; $g(1)=1$ since the point $(1,1)$ is on the graph of $g ; f^{\prime}(1)=2$ since the slope of the line segment between $(0,0)$ and $(2,4)$ is $\frac{4-0}{2-0}=2 ; g^{\prime}(1)=-1$ since the slope of the line segment between $(-2,4)$ and $(2,0)$ is $\frac{0-4}{2-(-2)}=-1$. Now $u(x)=f(x) g(x)$, so $u^{\prime}(1)=f(1) g^{\prime}(1)+g(1) f^{\prime}(1)=2 \cdot(-1)+1 \cdot 2=0$.
(b) $v(x)=f(x) / g(x)$, so $v^{\prime}(5)=\frac{g(5) f^{\prime}(5)-f(5) g^{\prime}(5)}{[g(5)]^{2}}=\frac{2\left(-\frac{1}{3}\right)-3 \cdot \frac{2}{3}}{2^{2}}=\frac{-\frac{8}{3}}{4}=-\frac{2}{3}$

## Page 1

