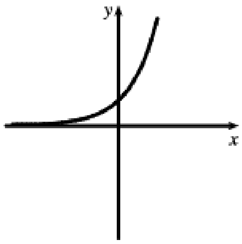
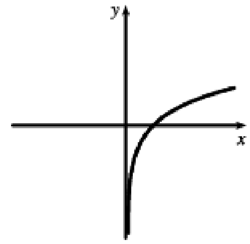
4. (a)



(b)



(c) In part (a), the graph of $y = e^x$ is a curve whose slope is always positive and increasing. In part (b), the graph of $y = \ln x$ is a curve whose slope is always positive and decreasing.

8. (a) If the position function is increasing, then the particle is moving toward the right. This occurs on t-intervals $(0, 2)$ and $(4, 6)$. If the function is decreasing, then the particle is moving toward the left—that is, on $(2, 4)$.

(b) The acceleration is the second derivative and is positive where the curve is concave upward. This occurs on $(3, 6)$. The acceleration is negative where the curve is concave downward—that is, on $(0, 3)$.

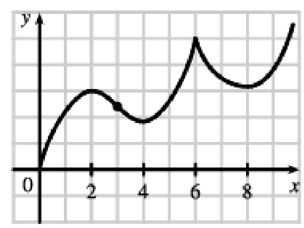
11. (a) f is increasing where f' is positive, that is, on $(0, 2)$, $(4, 6)$, and $(8, \infty)$; and decreasing where f' is negative, that is, on $(2, 4)$ and $(6, 8)$.

(b) f has local maxima where f' changes from positive to negative, at $x = 2$ and at $x = 6$, and local minima where f' changes from negative to positive, at $x = 4$ and at $x = 8$.

(c) f is concave upward (CU) where f' is increasing, that is, on $(3, 6)$ and $(6, \infty)$, and concave downward (CD) where f' is decreasing, that is, on $(0, 3)$.

(d) There is a point of inflection where f changes from being CD to being CU, that is, at $x = 3$.

(e)



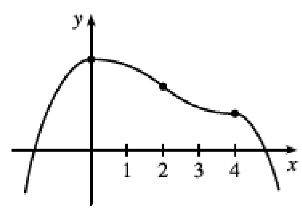
15. $f'(0) = f'(4) = 0 \quad \Rightarrow \quad$ horizontal tangents at $x = 0, 4$.

$f'(x) > 0$ if $x < 0 \quad \Rightarrow \quad f$ is increasing on $(-\infty, 0)$.

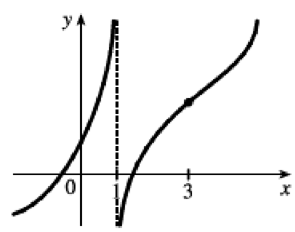
$f'(x) < 0$ if $0 < x < 4$ or if $x > 4 \quad \Rightarrow \quad f$ is decreasing on $(0, 4)$ and $(4, \infty)$.

$f''(x) > 0$ if $2 < x < 4 \quad \Rightarrow \quad f$ is concave upward on $(2, 4)$.

$f''(x) < 0$ if $x < 2$ or $x > 4 \quad \Rightarrow \quad f$ is concave downward on $(-\infty, 2)$ and $(4, \infty)$. There are inflection points when $x = 2$ and 4.



16. $f'(x) > 0$ for all $x \neq 1$ with vertical asymptote $x = 1$, so f is increasing on $(-\infty, 1)$ and $(1, \infty)$. $f''(x) > 0$ if $x < 1$ or $x > 3$, and $f''(x) < 0$ if $1 < x < 3$, so f is concave upward on $(-\infty, 1)$ and $(3, \infty)$, and concave downward on $(1, 3)$. There is an inflection point when $x = 3$.



21. (a) Since e^{-x^2} is positive for all x, $f'(x) = xe^{-x^2}$ is positive where $x > 0$ and negative where $x < 0$. Thus, f is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$.

(b) Since f changes from decreasing to increasing at $x = 0$, f has a minimum value there.

23. (a) To find the intervals on which f is increasing, we need to find the intervals on which $f'(x) = 3x^2 - 1$ is positive.
$3x^2 - 1 > 0 \quad \Leftrightarrow \quad 3x^2 > 1 \quad \Leftrightarrow \quad x^2 > \frac{1}{3} \quad \Leftrightarrow \quad |x| > \sqrt{\frac{1}{3}}$, so $x \in \left(-\infty, -\sqrt{\frac{1}{3}}\right) \cup \left(\sqrt{\frac{1}{3}}, \infty\right)$. Thus, f is increasing on $\left(-\infty, -\sqrt{\frac{1}{3}}\right)$ and on $\left(\sqrt{\frac{1}{3}}, \infty\right)$. In a similar fashion, f is decreasing on $\left(-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}\right)$.

(b) To find the intervals on which f is concave upward, we need to find the intervals on which $f''(x) = 6x$ is positive.
$6x > 0 \quad \Leftrightarrow \quad x > 0$. So f is concave upward on $(0, \infty)$ and f is concave downward on $(-\infty, 0)$.

(c) There is an inflection point at $(0, 0)$ since f changes its direction of concavity at $x = 0$.